Combustion Instability
Screech in Gas Turbine Afterburners

A Thesis
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by
Kampa Ashirvadam

Department of Aerospace Engineering
Indian Institute of Science
Bangalore - 560012

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Abstract

Gas turbine reheat thrust augmenters known as afterburners are used to provide additional thrust during emergencies, take off, combat, and in supersonic flight of high-performance aircrafts. During the course of reheat development, the most persistent trouble has been the onset of high frequency combustion instability, also known as screech, invariably followed by rapid mechanical failure. The coupling of acoustic pressure upstream of the flame stabilizer with in-phase heat-release downstream, results in combustion instability by which the amplitude at various resonant modes — longitudinal (buzz — low frequency), tangential or radial (screech — high frequency) – amplifies leading to deterioration of the afterburner components.

Various researchers in early 1950s have performed extensive testing on straight jet afterburners, to identify screech frequencies. Theoretical and experimental work at test rig level has been reported in the case of buzz to validate the heat release combustion models. In this work, focus is given to study the high frequency tangential combustion instability by vibro-acoustic software and the tests are conducted on the scaled bypass flow afterburner for confirmation of predicted screech frequencies.

The wave equation for the afterburner is solved taking the appropriate geometry of the afterburner and taking into account the factors affecting the stability. Nozzle of the afterburner is taken into account by using the nozzle admittance condition derived for a choked nozzle. Screech liner admittance boundary condition is imposed and the effect on acoustic attenuation is studied. A new combustion model has been proposed for obtaining the heat release rate response function to acoustic oscillations. Acoustic wave – flame interactions involve unsteady kinetic, fluid mechanic and acoustic processes over a large range of time scales. Three types of flow disturbances exist such as : vortical, entropy, and acoustic. In a homogeneous, uniform flow, these three disturbance modes propagate independently in the linear approximation. Unsteady heat release also generates entropy and vorticity disturbances. Since flow is not accelerated in the region of uniform area duct, vortical and entropy disturbances are treated as insignificant, as these disturbances are convected out into atmosphere like an open-ended tube, but these are considered in deriving the nozzle admittance condition. Heat release fluctuations that arise due to fluctuating pressure and temperature are taken into consideration. The aim is to provide results on how flames respond to pressure disturbances of different amplitudes and characterised by different length scales. The development of the theory is based on large activation energy
asymptotics. One-dimensional conservation equations are used for obtaining the response function for the heat release rate assuming the laminar flamelet model to be valid. The estimates are compared with the published data and deviations are discussed.

The normalized acoustic pressure variation in the afterburner is predicted using the models discussed earlier to provide an indication of the resonant modes of the pressure oscillations and the amplification and attenuation of oscillations caused by the various processes. Similar frequency spectrum is also obtained experimentally using a test rig for a range of inlet mean pressures and temperatures with combustion and core and bypass flows simulated, for confirmation of predicted results.

Without the heat source only longitudinal acoustic modes are found to be excited in the afterburner test section. With heat release, three additional tangential modes are excited. By the use of eight probes in the circumferential cross section of afterburner it was possible to identify the tangential modes by their respective phase shift in the experiments.

Comparison of normalized acoustic pressure and phase with and without the incorporation of perforate liner is made to study the effectiveness of the screech liner in attenuating the amplitude of screech modes. By the analysis, conclusion is drawn about modes that get effectively attenuated with the presence of perforate liner. Parametric study of screech liner porosity factor of 1.5 % has not shown appreciable attenuation. Whereas with 2.5 % porosity significant attenuation is noticed, but with 4 % porosity, the gain is very minimal. Hence, the perforate screech liner with the porosity of 2.5 % is finalized.

From the rig runs, first pure screech tangential mode and second screech coupled tangential modes are captured. The theoretical frequencies for first and second tangential modes with their phases are comparable with experimental results. Though third tangential mode is predicted, it was not excited in the experiments. There was certain level of deviation in the prediction of these frequencies, when compared to the experimentally obtained values. For this test section of length to diameter ratio of 5, no radial modes are encountered both in the analysis and experiments in the frequency range of interest.

In summary, an acoustic model has been developed for the afterburner combustor, taking into account the combustion response, the screech liner and the nozzle to study the acoustic instability of the afterburner. The model has been validated experimentally for screech frequencies using a model test rig and the results have given sufficient confidence to apply the model for full scale afterburners as a predictive design tool.
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Nomenclature

$\tilde{p}$  Weighing function, page 20

$\{Q_i\}$  Acoustic excitation vector, see equation (3.29), page 21

$q$  Dimensional heat release rate, see equation (3.8), page 18

$q^*$  Volumetric heat release rate per unit time (J/m$^3$ s)

$R$  Specific gas constant for the gas (J/kg K)

$s$  Entropy per unit volume (J/m$^3$ K)

$t$  Dimensionless time, see equation (3.8), page 18

$T^*$  Absolute temperature (K)

$t^*$  Time (s)

$\text{thk}$  Thickness of the perforate (m)

$u$  Dimensionless velocity, page 18

$u^*$  Velocity (m/s)

$\gamma$  Volume of the domain, page 20

$w_f$  Chemical reaction rate of fuel

$\Phi$  Porosity

$\gamma$  Specific heat ratio

$\lambda$  Flame speed eigen value, see equation (4.44), page 34

$\mu$  Dynamic viscosity (N s/m$^2$)

$\nu$  Stoichiometric air to fuel ratio

$\Omega$  Boundary of the domain

$\omega$  Angular frequency

$\dot{\Omega}_f$  Reaction rate of the fuel per unit surface area of the flame, see equation (4.4), page 30

$\dot{\omega}_f$  Turbulent reaction rate of fuel (kg/ m$^3$ s)

$\Omega_p$  Part of the surface where pressure is specified

$\Omega_v$  Part of the surface where velocity is specified

$\Omega_Z$  Part of the surface where impedance is specified

$\theta_w$  Characteristic wave propagation time, see equation (3.8), page 18

$\rho^*$  Density (kg/m$^3$)
\( \sigma \)  Flame surface area per unit volume
\( \tau \)  Non-dimensional temperature, page 32
\( \Theta \)  Non-dimensional overall activation energy, see equation (4.44), page 34
\( \theta \)  Non-dimensional time, page 32
\( \xi \)  Non-dimensional space variable, page 32
Chapter 1

Introduction

1.1 Screech in afterburners

Gas turbine reheat thrust augmenters known as afterburners are used to provide additional thrust during emergencies, take off, combat, and in supersonic flight of high-performance. Afterburners provide a lightweight, low-capital cost method to greatly increase engine thrust. During the course of reheat development, the most persistent trouble has been the onset of a high frequency screech. It is characterized by a peculiar violence, and its onset is invariably followed by rapid mechanical failure. This failure evinces itself in the tearing of the sheet metal, or if the screech is mild, persistent breakage of bolts or slackening the nuts [1].

Two types of instabilities are encountered in afterburners. They are buzz and screech. Reheat buzz is a low frequency, self-excited oscillation that can occur above a certain fuel-air ratio. Screech is accompanied by high frequency pressure oscillations that may be of such magnitude as to cause rapid deterioration of the burner. Screech might be, or closely related to, some form of resonant oscillation or also known as flame-driven resonant oscillations or combustion instability. The afterburner-inlet conditions at which screech occur differs widely for various afterburner designs. Combustion-driven flow oscillations that arise in combustors and the afterburners are difficult to predict [2].

Because of destructive nature of screeching combustion considerable effort is required to find methods of mitigating or preventing the occurrence of screech. Screech is associated with transverse oscillations. It is reported that the perforated liners are effective in mitigating transverse oscillations over the full operable range of fuel-air ratio for burner [3].

1.2 Objectives of the Thesis

The screech combustion instability in real afterburners pose several complexities to analyse theoretically such as variable area cross section, mean flow through core and bypass and modelling of combustion source. With a few simplifications and without compromising on the essential issues, theoretical estimates are carried out by a three dimensional finite element analysis by considering the combustion source and perforate liner screech damper. Then, the actual configuration of
1.2. Objectives of the Thesis

The afterburner is tested for confirmation of identified screech frequencies and that their amplitudes do not overshoot the permissible limits for the safe afterburner operation.

In this work, with the help of available vibro-acoustic software theoretical analysis for the identification of screech frequencies is undertaken in an afterburner in the presence of combustion source oscillations. The resonant acoustic pressure downstream is coupled with heat release upstream of the flame stabilizer. Thus screech phenomenon is a coupled combustion instability problem. A new combustion model has been proposed for obtaining the heat release rate response function. Most of the previous work is focused on, the heat release response functions which are based on mixture ratio variation and / or flame surface area variation. The earlier theoretical estimates are for the longitudinal mode of oscillations at relatively low frequencies. These models are not applicable to the current work, where transverse modes at high frequencies (up to about 2000 Hz) are dealt with. Since pressure directly affects the reaction rates, the classical laminar flame approach has been adapted for developing the heat release response function. The combustion related sound pressure level as a function of frequency is applied at various locations in the afterburner flame stabilizer zone and the solution is obtained. The combustion oscillations are expected to amplify the screech frequencies, whereas the presence of perforate liner attenuates the same by certain level. The normalized acoustic pressure variation along the circumference of the test section for the inlet harmonic particle excitation, combustion sources distributed in the plane of flame stabilizer for a frequency range required to be predicted to provide an indication of circumferential resonant frequencies and their phase shift. Similar frequency spectrum is obtained for the acoustic pressure variation by the rig testing for a given varied inlet mean pressure and temperature with afterburner combustion with core and bypass flows simulated for confirmation of predicted results.

By the predicted acoustic pressure history along the test section and at given cross-section of the test section the screech modes are identified as, whether it is pure screech mode or coupled mode. The relative nodal content of the acoustic pressure amplitude is established by plotting the data in the circumferential direction at two stations namely front and back of the test section.

Comparison of normalized acoustic pressure and phase with and without the incorporation of perforate liner is made to study the effectiveness of the screech liner in attenuating the amplitude of screech modes. By the analysis, conclusion is drawn about modes that get effectively attenuated with the presence of perforate liner.

Much of the research work reported in the open literature is confined to reheat buzz that is of low frequency combustion instability of longitudinal modes, at rig testing level. Earliest engine manufacturers have addressed the screech frequency problem in straight jet configurations of afterburners by expensive full scale testing. It is noted that the screech frequency attenuation is mission critical for the combat military aircraft. To circumvent high cost, time and hazardous potential involved in addressing screech problem, the available vibro-acoustic software and \( \frac{1}{3} \)rd engine mass flow test rig are employed to gain clear understanding of the
1.3 Description and operation of the afterburner

Afterburner is an exhaust system that is fitted to the exit of low pressure turbine in the gas turbine engine. The main components of afterburner are: diffuser that reduces the flow velocity, screech liner to attenuate the transverse oscillations, fuel manifolds to provide proper fuel distribution and a flame stabilizer that provides the recirculation zone for flame anchoring. Figure 1.1 shows the construction of afterburner. Afterburner receives vitiated air from the exit of low pressure turbine (LPT) at a temperature range of 780 K to 1065 K, and a pressure range of 55 kPa to 380 kPa from high altitude operation to sea level. The exit velocity of LPT is of the order 0.5 to 0.6 Mach. When one desires to light up afterburner by admitting fuel into the core cavity, it is extremely difficult or impossible to achieve a stable flame due to high incoming velocity. Hence, a diffuser is employed to reduce the velocity to the level of 0.3 Mach. A recirculation zone is created by a V-shaped bluff body. Fuel is admitted through the manifold which is situated upstream of central V-shaped-flame-stabilizer. The catalyst (platinum-rhodium) which is located in the flame stabilizer helps in igniting the mixture of hot air from LPT and the fuel that is sprayed in the vaporized form through the manifold at a pressure of 700 kPa. In the experimental test facility hot air is provided by a pre-heater ahead of afterburner. At about 450°C combustion is initiated. Then, all the other three manifolds are activated to achieve full afterburning at a fuel manifolds pressure of 2720 kPa. To simulate afterburner entry conditions by the pre-heater, a fuel / air ratio in the range of 0.015 to 0.02 is needed. For full afterburning the overall fuel air ratio is in the range of 0.05 to 0.06.
1.4 Organization of the thesis

Chapter 2 gives a review of combustion instability phenomenon, amplification mechanisms and attenuation, combustion source models and subsequently research carried out on afterburner systems in the past. Chapter 3 gives theoretical formulation of the afterburner configuration that was used in the FEM analysis through SYSNOISE software. The coupled governing equations for acoustic analysis and combustion source oscillations are derived from fundamentals. Applicable boundary conditions are presented. Certain simplifications are made while preparing the mesh from the actual geometry. The centre body, struts, and variable area of the test section are considered along with the perforated liner in the geometrical modelling and a three dimensional tetrahedron mesh is created for the purposes of analysis. The medium is considered to be with uniform acoustic properties. The other approximations and the limiting conditions appropriate to various regions are discussed.

A new combustion model has been proposed for obtaining the heat release rate response function to acoustic oscillations. Acoustic wave - flame interactions involve unsteady kinetic, fluid mechanic and acoustic processes over a large range of time scales. The development of the theory is based on large activation energy asymptotics. One-dimensional conservation equations are used for obtaining the response function for the heat release rate assuming the laminar flamelet model to be valid. These are discussed in Chapter 4.

Chapter 5 presents the afterburner test facility description, capability, operation and the tests carried out on the (1/3)rd scale afterburner. The data is acquired by the dynamic pressure probes. Chapter 6 gives the discussion on theoretical and experimental results. Chapter 7 gives the conclusions along with limitations in the analysis, and difficulties encountered during experiments and the expertise obtained in addressing screech related problems in the afterburners.
Chapter 2

Literature Survey

Combustion instabilities arise at different stages of combustion process. The characteristics of such oscillatory instabilities are briefly discussed in section 2.1. Mechanism of amplification of oscillatory combustion instability is presented and then, in section 2.2, the specific issue of instability problem in afterburners is presented. Finally in section 2.3 the present problem definition and methodology followed are discussed along with complexities involved in theoretical analysis.

2.1 Characteristics of oscillatory instabilities

The combustion instabilities are summarized into three types [4] such as:

1. Intrinsic instabilities which are inherent to the combustion and fluid physics,

2. Chamber instabilities which result from the interaction of the combustion process with a combustion chamber, and

3. System instabilities that involve the interaction of the combustion processes in a chamber with upstream feed lines and / or downstream exhaust.

In each of these three categories, different physical processes may contribute to the instability. Another categorization of instabilities is in terms of the physical processes involved. For example, there are buoyant instabilities, hydrodynamic instabilities and acoustic instabilities, among others.

The oscillatory instability in gas-turbine combustors as well as afterburners arise from the coupling of unsteady heat release with acoustic waves in a chamber, resulting in repeated pressure fluctuations at various characteristic frequencies. The instability frequencies are associated with the geometry of the device and may be influenced by interactions between the device and flow field. The interactions causing these self-excited oscillations are complex because of the coupling of the flow field with unsteady (and highly non-linear) heat release. Some experiments have been interpreted [5,6] to indicate that a primary cause for generation of instabilities is an acoustic wave generated by unsteady heat release that trips a Kelvin-Helmoltz instability in the flow [7], where high density gradients, shear, and substantial vortices exist. The instability modifies both the overall flame
structure and flow (turbulence), and hence an effective closed-loop feedback system is generated.

In 1878 Rayleigh proposed a criterion that has evolved into a clear rule for the potential amplification of an acoustic wave in combustion system, essentially that positive correlation of the heat-release and pressure variations over the period of one acoustic cycle results in amplification of oscillations [8],

$$\int_0^T p'(t)q'(t) \, dt > 0 \quad (2.1)$$

where $p'$ and $q'$ are the pressure and heat-release perturbations, respectively, as function of time $t$, and $T$ denotes the period of the oscillation. Unfortunately, it is difficult to apply these criteria in a practical setting, as may be observed in studies [9,10,11] of one-dimensional systems designed to model reheat buzz.

Several mechanisms have been identified that contribute to acoustic instability amplification, some of which are:

1. Air/fuel ratio fluctuations which was recently investigated by Lieuwen and Cho [12] resulting in heat-release perturbations at the flame can cause acoustic waves to propagate upstream into the feed lines and cause perturbations in the incoming air/fuel mixture. These perturbations may be carried by the mean flow and trigger a fluctuation at the base of the flame, closing the instability loop. Several studies have addressed this possible mechanism. For example, Sacarini and Freeman [13] studied the fuel-air fluctuations in a simple duct and concluded that the strong potential of these fluctuations to drive instabilities justified substantial effort in mitigating air/fuel fluctuations. In their work [14], Sacanini et al [14] defined a parameter $\sigma$ as an indicator of the efficiency of mixing duct,

$$\sigma = \frac{\phi' \phi}{u'} \quad (2.2)$$

where $\phi'$ is the ratio of air/fuel-ratio perturbations to the mean air/fuel-ratio, and $u'$ is the ratio of velocity perturbations to the mean flow velocity. Their work focused on trying to get this parameter as close to zero as possible, which was accomplished by improving the mixing quality of the reactants by using multiple fuel-injectors.

2. Vortices shed from the flame holder were suggested as a cause of combustion instabilities as early as 1956 by Rogers and Marble [15]. More recently, experimental investigation by Poinsot et al [5] looked at this as a possible source of combustion instability. The instability is triggered when the vortices shed at the flame holder entrain unburned mixture, which propagate downstream and causes a sudden heat release at some point downstream. This triggers an acoustic wave propagating upstream, closing the feedback loop. Culick and Magiawalla [16] investigated vortices shed from the flame holder consisting of pure products, which would impinge on obstacles downstream (e.g. the nozzle) and cause pressure oscillations to intensify. This
2.2 Combustion instabilities in afterburners

The combustion instabilities that are encountered in the Gas Turbine afterburners are categorized into buzz and screech. Buzz is low frequency combustion instability involving the propagation of low frequency longitudinal acoustic waves in the duct, whereas screech is associated with high frequency tangential acoustic waves. Quite a few authors have attempted to address the problem of buzz by analytical methods and experiments in laboratory test rigs. In this section, research pertaining to low frequency combustion instability is discussed.

Dowling and Bloxidge [21] were the first to develop a theory to describe the onset of buzz in ducts with simple geometry and to determine the frequency of unstable oscillations. This paper is discussed in detail as it formed basis for all subsequent investigators who worked in the area of low frequency combustion oscillations. The authors have drawn a conclusion from the schlieren photographs that the longitudinal pressure waves perturbed the flame, causing the flame to move and change in flame surface area, thereby altering the instantaneous heat

investigation was of purely acoustic phenomenon with no heat-release contribution. Mateev and Culick [17] recently investigated the formation of vortices behind the flame holders and their interaction with flow-field perturbations in premixed combustors, using a newly developed quasi-steady model. In this model, they addressed a dump combustor in which they assumed constant fluid properties, vortex burning as the only source of instability (without vortex-surface interaction), and vortex propagation at the mean flow velocity. They were able to partially validate the model against experimental results for vortex shedding in a non-reacting oscillating cold flow. The authors cited the concern that reacting flows might behave differently and that resulting vortex shedding and interaction in reacting flows might follow a different pattern.

3. The effect of entropy waves (the phenomenon of localized hot spots) on flow-field instabilities was suggested in early work by Chu [18], who considered their influence on combustion instabilities to be minimal except at low frequencies. Polifke et al [19] state that since the hot spots are transported by the mean flow (usually at low velocity), effects of entropy waves have been assumed to exist at low frequencies [20].

Little work has been done to include damping effects in the body of literature on combustion instabilities. Williams [4] presented some of the possible damping sources in an oscillating combustor environment. They are wall damping, particle damping, relaxation damping, homogeneous damping, nozzle damping etc. When choked flow conditions are met, the propagation of acoustic waves is restricted at the nozzle and no propagation of longitudinal waves is allowed upstream from a diverging supersonic-flow section. This boundary can absorb energy depending on the flow conditions. Other types of damping are due to perforated walls and cooling flows near the wall, and heat addition mechanisms acting out of phase with pressure perturbations.
release rate. To model this phenomenon, a control volume analysis is performed by which flame zone perturbations in heat release rate are related to the upstream conditions. Equations for upstream and downstream condition of the flame for the perturbations of pressure, velocity, density and temperature are set up. Boundary conditions are: i) choked flow at entry, ii) Howes reflection coefficient criteria, iii) conservation relations across the flame. Third boundary condition accommodates, heat release where flame surface and velocity of the element of flame are taken into consideration. By the application of momentum conservation across the control volume, the form drag of flame stabilizer is considered. As combustion converts the chemical energy into tangible form, the linearized equation of energy conservation is written. The relationship between the unsteadiness in combustion and the velocity perturbation has been deduced by heuristic argument and the same is substituted in the energy equation. Thus six homogeneous equations for six unknowns are set up. The unknowns are the amplitudes of incident and reflected waves from flame at upstream and downstream, heat release rate and the acoustic wave amplitude of the convected hot spot. The sustaining unstable acoustic wave frequency is deduced for which the determinant of the above matrix vanishes. Computations are made for single unstable frequency for various equivalence ratios. The predicted buzz frequency is compared with experimental data. At the buzz frequency by numerical method heat release rate magnitude and phase are predicted. The author proposed that there is a need to refine the flame model that is proposed in the theory by experiments.

Bloxidge et al [10] have provided rigorous theoretical treatment to the above problem. By the support of extensive experimentation on the same test rig, the flame model is described. They have assumed the rate of heat release to be proportional to the measured light emission from radicals. Flame stabilizer blockage is neglected. By an iterative procedure buzz frequency is predicted and compared with the experimentally obtained value and there was very close agreement.

The influence of a twin stream supply on acoustically coupled combustion oscillations was investigated by Macquisten and Dowling [22]. The duct length was 0.95 m with rectangular cross section of 70×70 mm. Ethylene was used as the fuel. Inlet flow velocities are of order 0.14 Mach. Inlet temperature is 290 K for the bypass flow and 540 K for the core flow. Only the longitudinal low frequencies were captured. The fundamental frequency is about 109 Hz. The onset of low frequency combustion instabilities was predicted by linear stability analysis by having a relationship between velocity perturbations at the flame holder and instantaneous heat release rate, which was derived from schlieren photographs. Though the bypass flow was simulated, the two streams were mixed in front of the flame stabilizer, thus having the same temperature. The cross-section of the test section is rectangular. Only the longitudinal modes were studied. It is known that the transverse modes differ widely if the duct cross-section is rectangular instead of circular as in the normal case.

The influence of geometric and flow variables on combustion instability in confined flames stabilized behind a disk was examined by Sivasegaram et al [23]. Core and bypass flows were simulated. Length of the duct was 480 mm and the
diameter of the duct 80 mm. Propane was used as fuel. Mean velocities upstream of flame holder was in the range of 6.5 m/s to 11.5 m/s. Only oscillations that were associated with longitudinal wave frequencies of the order 150 Hz to 200 Hz ware investigated. The studies were conducted with and without exit nozzles. The mass flow conditions are quite small when compared with the present trend. The study has not included the presence of perforate liner.

Dowling [24] has summarized the work related to buzz in afterburners in The 1999 Lanchester lecture. The governing equation for coupled combustion instability is described by the following inhomogeneous wave equation 2.3,

\[
\frac{1}{c^2} \frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \left( \frac{\gamma - 1}{c^2} \right) \left( \frac{\partial q'}{\partial t^*} \right)
\]  

(2.3)

The term on the right hand side describing how the unsteady addition of heat generates pressure disturbances. The status of the afterburner can be summarized as follows.

1. When equation 2.3 reduces to homogeneous wave equation whose solutions give the usual organ pipe modes. For the one dimensional case, by the method of separation of variables in the form of 

\[
p'(z,t) = F(t)G(z),
\]

with the boundary condition of zero pressure variation at \( z = 0 \) and \( z = l \) leads to 

\[G(z) = \sin(n\pi z/l)\]

and 

\[F(t) = F_0 \sin(\omega n t + \phi)\]

where \( \omega_n = n\pi c^*/l \), \( n \) is an integer, and \( F_0 \) and \( \phi \) are constants determined by the initial conditions. These solutions represent the normal organ pipe modes of oscillation with the pressure fluctuates without decay at the resonant frequencies \( n\pi c^*/l \).

2. A few combustion models are discussed to bring out the influence of unsteady heat release rate \( q'(z,t) \) on the above pressure perturbations. One model suggested has the form \( q' = -\alpha c^2 \gamma^{-1} p' \), where \( \alpha \) is a complex quantity which accounts for the phase difference between the pressure and heat release perturbations. The boundary conditions ensure that \( G(z) \) has the same form and the solution is given by \( p'(z,t) = F(t) \sin(n\pi z/l) \). By the substitution into Eq. 2.3, 

\[
\frac{1}{c^2} \ddot{F} + \alpha \dot{F} + \frac{n^2 \pi^2}{l^2} F = 0
\]

is obtained which describes a damped oscillator. If \( p' \) and \( q' \) are 180° out of phase, the constant \( \alpha \) is real and positive and describes damped oscillations. If, however, \( p' \) and \( q' \) are in phase \( \alpha \) is real and negative and the solution grows in time. Thus Rayleigh criterion is recovered. Heat addition in phase with pressure perturbation is required for energy input into acoustic waves.

3. The second model suggested is: \( q' = \frac{\beta}{c^2} \frac{\partial p'}{\partial t} \). By the substitution into governing inhomogeneous equation, 

\[
\frac{(1-\beta)}{c^2} \ddot{F} + \frac{n^2 \pi^2}{l^2} F = 0
\]

is obtained. This has no damping term, instead the interaction between heat release and pressure leads to a frequency shift. If \( q' \) leads \( p' \) by \( \frac{1}{4} \) cycle, \( \beta \) is positive and frequency increases. If, however, \( q' \) lags \( p' \) by \( \frac{1}{4} \) cycle, \( \beta \) is negative and the frequency decreases.

4. The examples considered above are over simplification of what occurs in practice. Bloxidge et al [10] studied the coupled combustion instability
by extensive experiments over a wide range of equivalence ratios. The distribution of heat release was determined through measurements of the light emission from $C_2$ radicals. They obtained an empirical correlation of the form

$$\frac{\dot{q}(z)}{\bar{q}(z)} \approx \frac{1}{1 + j\omega(2\pi r/\bar{u})} \frac{\hat{u}}{\bar{u}}$$

(2.4)

The flame model suggested in this work is purely empirical and was deduced by simply expressing experimental results in an appropriate non-dimensional form.

5. Dowling [25] has developed a kinematic flame model, in which flame surface area propagates with turbulent burning velocity, $S_u$, and

$$\frac{\dot{q}(z)}{\bar{q}(z)} = f\left(\frac{u'}{\bar{u}}, S_u\right).$$

6. Brookes et al [26] studied flame response through CFD by simulating the results obtained by Bloxidge et al [10]. CFD model is based on laminar flamelets, the reaction rate being obtained from the product of laminar flame speed and a flame surface density function.

7. Fannin [27] has developed coupled combustion oscillation model. He performed control volume analysis in order to determine how the velocity perturbations in the acoustic field would affect the overall heat release rate. The unsteadiness is described by conservation of species and energy in the control volume and the heat release rate was computed as the change in sensible enthalpy from the reactant flow entering to products exiting the control volume. The species consist of fuel, air, and products. The volume is assumed to be steady, but the mass fraction of species and temperature of the control volume are allowed to vary. Single step Arhenius kinetics describe the rate of change of reactant and products species in the control volume. The species are assumed to be perfectly and instantaneously mixed.

As early as 1950s, extensive research have been conducted in the area of screech through expensive experiments, that has appeared in the literature.

Edwards [1] has reported the occurrence of screech in the tests conducted at the National Gas Turbine Establishments. The amplitude of screech oscillations could be as high as 40 kPa. And the fundamental screech modes range between 250 Hz to 750 Hz and 450 Hz to 1300 Hz for the first harmonic transverse modes depending upon the temperature of the gases.

Lundin et al [3] have conducted experiments on straight jet afterburners whose inlet velocities are in the rage of 0.32 Mach, and inlet pressure 150 kPa. A perforated liner with 4.75 mm diameter holes was installed concentrically with the afterburner. Fuel injection was by 24 spray bars, and flame stabilization was achieved with the help of two concentric V-shaped stabilizers. Two configurations are considered with L/D ratio of combustion volume 2.3 and 4.2. A screech frequency of 650 Hz was encountered. The first and the fourth transverse modes were encountered. Screech holes were spread up to 915 mm from the flame holder. With the perforated liner, screech was completely eliminated. It may be
noted here that straight jet engines are replaced by small bypass engines and the latest engines operate at much higher pressures and in the absence of theoretical estimates, a lot of effort and time are required in the experiments to identify the screech frequencies.

The investigation of combustion instability was conducted by James et al [28] in two separate test facilities, a 26 inch-diameter duct rig and a 32-inch-diameter short afterburner attached to a full-scale turbojet engine. Spray bars and diametrical flame holder are used. Exit nozzle was not fitted and bypass flow was not simulated in the rig testing. Inlet temperature is 950 K. L/D of combustion volume is 2.8. First transverse mode of oscillation occurred at 635 Hz. In the 32 inch diameter afterburner a perforated liner was used. The bypass air around the perforated liner was also at 950 K. In this case bypass and core flow are at the same temperature. Several configurations were tested in the rig.

The Lewis Laboratory staff [2] have summarized all the preliminary investigations in the area of combustion screech. All of the work is based on experiments. Several straight jet afterburner configurations data was presented. It is observed that without the perforated screech liner, at all equivalence ratios, screech commenced. With the perforated liner, screech occurred only very close to stoichiometric condition or eliminated completely.

There was a long silence on the subject after the 1950s. Increasing demands for higher afterburner performance of late have required operation at progressively higher fuel-air ratios, which has increased the occurrence and intensity of screeching combustion.

Much of earlier works primarily dealt with reheat buzz that occur at low frequencies and tested at low capacity test rigs. In these tests, the actual afterburner configurations were not considered. Though full size afterburners for screech were investigated in 1950s, they were straight jet low thrust engines without bypass flow. Though every engine manufacturer studies the problem of screech, before finalization of the design, the information remains classified. Thus there is a significant requirement of performing analysis á priori to identify screech frequencies in real afterburners and provide a method to mitigate screech that could help in the finalization of the afterburner design for the modern gas turbine engines. Having theoretical estimates, one can undertake only confirmatory experimental tests and save a lot of time and effort.

Main conclusions drawn from the above study of literature are as follows:

1. Combustion instability problem is dealt for low frequency longitudinal modes of oscillations namely reheat buzz by theory and test rig level at very low flow conditions.

2. Combustion models developed are based on flame surface area and velocity fluctuations, which are at best valid under quasi-steady conditions.

3. In the area of screech, which is a high frequency transverse oscillation, only experimental data is reported for the straight jet engines. No rigorous theoretical analysis is reported.

Pressure wave-flame interaction is a very important phenomenon in afterburners due to flame acceleration and turbulent break up of laminar flames.
2.2. Combustion instabilities in afterburners

A comprehensive review of physics of oscillatory combustion in Gas Turbines is well documented by Ibrahim et al. [29]. Besides describing the characteristics, amplification, damping of oscillatory instabilities, the authors suggested possible methods of theoretical study including unsteady computational fluid dynamics simulation. Hathout [30], Ducruix et al [31], Keller [32], Bellucci et al [33], Dowling and Stow [34], Searby and Rochwerger [35], Culick [36], Lee and Santavicca [37], Lieuwen [38], and Fleifil [39] are some of the main authors who have explained the phenomenon of pressure wave-flame interactions leading to combustion instability elaborately. In this regard, Jean Pierre has derived Raleigh criterion analytically. Ducruix has derived governing equations for combustion instability by incorporating reaction kinetics due to heat release. However no estimates are made. Dowling has suggested few simple models for heat release rate fluctuation computation.

Particular interest in the present work is to consider chemical kinetics of reaction in the combustion zone and to see how acoustic pressure couple with heat release fluctuations in the frequency range of 100–1500 Hz.

Clarke [40] discussed the pressure flame interaction at more fundamental level. The work that is having lot of similarities with present thesis in respect of acoustic combustion source characterization is due to McIntosh [41]. Using time and length scale ratios McIntosh gave mathematical treatment for modelling combustion heat release response function. Acoustic wave lengths of interest in combustion systems is in the range of $10^{-2}$ to $10^{0}$ m, flame response time scale is $10^{-3}$ to $10^{-2}$ s [42]. Mach number, $M$ is of the flame burning velocity of the order $10^{-4}$ and $\Theta$ is of the order 10 (dimensionless overall activation energy).

McIntosh [41] has proposed three distinct cases of pressure - premixed flame interactions based on the time scales of the various processes.

1. Low amplitude coupling ($\delta \sim O(M)$, time scale $\approx 1$ i.e. medium wave length of the order 3500 Hz): In this the assumption is coupling is through velocity perturbation. This is significant only when heat loss comes into effect. In afterburners that is not relevant. Moreover such low level acoustics simply regarded the flame as a contact discontinuity swept along with a small unsteady acoustics. In such case pressure gradients do not exist in the combustion zone. That is to say that inner reaction zone is not affected by pressure field. The effect of presence of disturbances is predominantly in the outer combustion zones which are swept away. Under the assumption of small amplitude, $\delta \sim O(M)$, there was no coupling of pressure changes with the temperature disturbances at the flame.

2. When $\delta$ is raised to the level of $\Theta^{-1}$ coupling can occur. For large wave lengths (time scale $<< 1$) van Harten et al [43] included density effects and for medium wave lengths (time scale $\approx 1$) density effects were not included. Here also no pressure gradients are felt in the combustion zone. The afterburner combustion experiences large wavelength (buzz) and medium wave length (screech) combustion instability.

3. Very high frequency waves: (time scale $\approx M^{-1}$) In such cases for large amplitudes of coupling ($\delta \approx \Theta^{-1}$), non linearities develop which are much
affected by the sharp density changes through the flame. Here pressure gradients are very important in the combustion zone which contains the full effect of any pressure wave passing through. It results in non-constant wave speed with non linearities for large amplitude disturbances.

McIntosh [41] has given formulation of above phenomenon by the equations of state, continuity, energy, species and momentum equations. The important assumption is density times thermal conductivity is a constant. It implies that the thermal conductivity is proportional to temperature which is experimentally found. This assumption also simplifies energy and species equations. By classical large activation energy analysis linear acoustic relations are derived for reaction zone and Eulerian equations for upstream and downstream of combustion zone. These equations are similar, but not identical to those of van Harten et al [43]. The major difference is that in their work, they assumed constant thermal conductivity and derived non linear partial differential equations, the non linearity arising out of the density effects. In McIntosh [41] work neither thermal conductivity nor density is assumed constant. By assuming harmonic inputs for temperature, chemical mixture concentration, mass flux and pressure, equations are linearized and then mass flux perturbation quantity was computed in the case of medium wave lengths for Lewis numbers 0.3, 0.5, 1.0 and 1.5. The main conclusion was that the fluctuations are roughly twice as large from the reaction zone side as those with unburnt upstream side.

In the present work, a new heat release response function due to acoustic pressure perturbations is developed from the fundamental equations and results are compared with the work of McIntosh [41] highlighting the similarities and differences in chapter 4.

2.3 Problem definition — Scope of analysis and testing

The real afterburners pose several complexities to analyze theoretically, such as variable area cross section, mean flow through core and bypass and unsteady response of combustion source. Early studies of acoustic wave oscillations through variable area ducts restricted their scope to the case of no mean flow. Several researchers have obtained the wave propagation solutions in ducts whose walls vary sinusoidally [44, 45, 46, 47, 48]. Others have used a discretization technique based on the solution for a single duct discontinuity [49, 50, 51]. The inclusion of the effects of a mean flow to provide more realistic model makes the problem considerably more difficult, and most studies have employed one or more simplifying assumptions. Some have used the assumption of one-dimensional flow and restricted their analysis to the propagation of the lowest acoustic mode through ducts with variable cross sections [52, 53]. However, the restrictions to propagation of the fundamental mode limit the usefulness of such analyses for realistic situations in which the sound is a combination of numerous modes (longitudinal, tangential and radial). For ducts with larger axial variations, coupling between modes will occur [54, 55, 56, 57].
Although there was considerable effort in the development of analytical and numerical methods to predict acoustic wave propagation in variable area ducts, combustion source modelling is not included especially for identifying the screech frequencies along with the influence of perforate liner to mitigate screech frequency amplitude. For this reason, theoretical estimates are carried out by a three dimensional finite element analysis by considering the combustion source oscillations and perforate liner. Figure 2.1 shows the block diagram in which, the methodology followed in the present work to study the high frequency combustion instability is presented. The governing equations describing the acoustic
2.3. Problem definition — Scope of analysis and testing

phenomenon due to combustion are derived. Finite element formulation is described along with boundary conditions. The equations are coupled with heat release. A new combustion model has been proposed for obtaining the heat release rate response function. As discussed earlier the heat release response functions currently explored in the literature are based on mixture ratio variation and/or flame surface area variation. These are quasi-steady models applicable to only low frequency oscillations.

Most of the previous work were focused on the longitudinal mode of oscillations at relatively low frequencies. These models are not applicable to the current work, which primarily deals with transverse modes at high frequencies (up to about 2000 Hz). Since pressure directly affects the reaction rates, the classical laminar flame approach has been adapted for developing the heat release response function. The combustion related sound pressure level as a function of frequency is applied at various locations in the afterburner flame stabilizer zone and the solution is obtained by available software. The coupling of flame and acoustic oscillations is expected to amplify the amplitude of pressure oscillations and the presence of perforate liner will attenuate the same by certain percentage. In the afterburners passive control of combustion instabilities is by the use of liners. Certain percentage of attenuation is achieved by drilling holes on the liner that will reduce acoustic energy from the combustion chambers that would otherwise return to the coupling mechanism of the feedback loop [58, 59, 60]. Mechel [61, 62] has suggested modelling methods to estimate perforate liner impedance. The complex liner impedance is modelled and implemented in the software where liner is located to assess the attenuation characteristics of combustion instabilities. Mechel [63] has provided a comprehensive review and compilation of the work on liners and has presented the formulae on different types of liners. The present work utilizes the formulae suggested in this reference to model the acoustic liner. The details of the theoretical formulation with appropriate boundary conditions are presented in chapters 3 and 4.

The actual configuration of afterburner was tested for confirmation of identified screech frequencies and that their amplitudes do not overshoot the permissible limits for the safe afterburner operation.
Chapter 3

Theory

In this chapter, the governing equations for the acoustic response of the afterburner by inclusion of combustion source are derived. Subsequently, boundary conditions, at inlet and at outlet of test section, acoustic impedance boundary condition that characterizes the screech liner, and exit nozzle admittance are presented. The Mathematical treatment is presented for completion.

3.1 The Governing Equations

The afterburner is essentially having two sections. In the first portion, the combustion process is considered as a reacting mixture flowing in a constant area duct with a flame anchored at a specific location in the duct. When the reactants are ignited, chemical energy is released in the form of heat, thus raising temperature of the flowing fluid. In the second section the burnt products are accelerated in the convergent nozzle to sonic velocity at throat. This type of combustion process sustains acoustic pressure and velocity oscillations. The flow in the afterburner, not only have inherent instabilities, but also respond readily to imposed fluctuations. Thus, the potential coupling between the unsteady components of pressure and heat release rate leads to their resonant coupling and growth. This phenomenon is referred to as thermoacoustic instability.

In the following section, equations governing the acoustic pressure and velocity are obtained in the presence of unsteady heat addition within the afterburner. These are manipulated to get the forced acoustic wave equation which is then reduced to the case when the mean heat release is negligible, that is, approximately homogeneous field, and when the mean flow is negligible.

The flow of gases in each of the two regions is governed by the well known Navier-Stokes equations. The basic assumptions used for the flow in the afterburner are:

1. three dimensional flow
2. inviscid flow, that is, the duct has negligible dissipation effect on the acoustic waves,
3. negligible thermal conduction to the surroundings.
Taking into consideration the assumptions above, the reactive gas dynamics are described using the vector conservation equations as:

1. **Mass continuity**
   \[
   \frac{\partial \rho^*}{\partial t^*} + \nabla \cdot (\rho^* \mathbf{u}^*) = 0 \quad (3.1)
   \]

2. **Momentum**
   \[
   \rho^* \frac{\partial \mathbf{u}^*}{\partial t^*} + \rho^* \mathbf{u}^* \cdot \nabla \mathbf{u}^* + \nabla p^* = 0 \quad (3.2)
   \]

3. **Energy**
   \[
   \rho^* \frac{\partial h^*}{\partial t^*} + \rho^* \mathbf{u}^* \cdot \nabla h^* - \left( \frac{\partial p^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla p^* \right) = q^* \quad (3.3)
   \]

where \( t^* \) is time, \( \rho^* \), \( \mathbf{u}^* \), \( p^* \) and \( h^* \) are the density, velocity vector, pressure, and specific enthalpy respectively, and \( q^* \) is the heat release per unit volume. The combustion products in the combustion zone behave as perfect gases governed by perfect gas state equation

\[
p^* = \rho^* R T^* \quad (3.4)
\]

where \( R \) is the gas constant and \( T^* \) the absolute temperature.

As is deduced from observations of the pressure recorded at any location in the combustion chamber, fluctuations are always present. Similar fluctuations of temperature, velocity, composition, etc., are also inevitable. An extremely turbulent condition is always present in the chamber, and is the cause for the intensity of the noise produced. When the frequency of gas oscillation in the combustion chamber is sufficiently high, the wavelength of standing wave oscillations are comparable to the duct dimensions and the gas pressure in the combustion chamber is not uniform at any instant in unsteady operation. In a region with heat input \( \rho^* \) varies through changes in both the pressure \( p^* \) and specific entropy \( s^* \). By chain rule of differentiation

\[
\frac{D \rho^*}{Dt^*} = \frac{1}{c^*} \frac{Dp^*}{Dt^*} + \frac{\partial \rho^*}{\partial s^*} \bigg|_p \frac{Ds^*}{Dt^*} \quad (3.5)
\]

where \( c^* \) is the speed of sound in the stagnant burnt gas. When viscous and heat conduction effects are neglected, \( \rho^* T^* \frac{Ds^*}{Dt^*} = q^* \). For a perfect gas

\[
\frac{\partial \rho^*}{\partial s^*} \bigg|_p = \frac{-\rho^*}{c_p^*} = -\frac{\rho^* T^*(\gamma - 1)}{c^*} \quad (3.6)
\]

where \( c_p^* \) is the specific heat at constant pressure. By noting \( \frac{D}{Dt^*} = \frac{\partial}{\partial t^*} + \mathbf{u}^* \cdot \nabla \) and \( \frac{Dh^*}{Dt^*} = T^* \frac{Ds^*}{Dt^*} + \frac{1}{\rho^*} \frac{Dp^*}{Dt^*} \), and by combining continuity and energy equation along with Eq. 3.5 and Eq. 3.6 the following equation is obtained.

\[
\left( \frac{\partial \rho^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla p^* \right) + \rho^* c^* \nabla \cdot \mathbf{u}^* = (\gamma - 1)q^* \quad (3.7)
\]
The above equations can be applied to a combusting gas, assuming that the reactants and products behave as perfect gases and that there is no molecular mass change during chemical reaction.

The dimensionless heat release, time, velocity, gas pressure, and density are defined as

\[ q = \frac{q^*}{c_p^* T_0^* \rho_0^*}; \quad \theta_w = \frac{L}{c^*}; \quad t = \frac{t^*}{\theta_w}; \quad p = \frac{p^*}{p_0^*}; \quad \rho = \frac{\rho^*}{\rho_0^*} \]  

(3.8)

where \( \theta_w \) is the characteristic wave propagation time required for a sound wave to travel a distance of \( L \). The velocity \( u^* \) of mass motion is expressed as a fraction of \( c^* \) and therefore \( u = u^* / c^* \). Thus in non-dimensional form the unsteady conservation equations are

\[
\frac{\partial \rho}{\partial t} + u \cdot \nabla \rho + \rho \nabla \cdot u = 0 \tag{3.9}
\]

\[
\frac{\partial u}{\partial t} + \rho u \cdot \nabla u + \frac{1}{\gamma} \nabla p = 0 \tag{3.10}
\]

\[
\frac{\partial p}{\partial t} + u \cdot \nabla p + \gamma \rho \nabla \cdot u = \gamma q \tag{3.11}
\]

In order to examine the onset of instability due to the small perturbations, the perturbation method is used to linearize the system of equations. The variables are separated into their mean (function of space only) and small perturbation (function of space and time) components as \( p = \bar{p} + p', \ u = \bar{u} + u', \ \rho = \bar{\rho} + \rho', \ q = \bar{q} + q' \), where \( \bar{\cdot} \) and \( \cdot' \) are the mean and the perturbation of a variable, respectively. The order of magnitude of terms are assumed to be \( \bar{p}, \bar{\rho}, T \sim 1 \) and \( p', u', \bar{u}, q' \ll 1 \).

The conservation of mean flow, which is assumed to be steady, can be described as

\[
\bar{u} \cdot \nabla \bar{p} + \bar{\rho} \nabla \cdot \bar{u} = 0 \tag{3.12}
\]

\[
\bar{\rho} \bar{u} \cdot \nabla \bar{u} + \frac{1}{\gamma} \nabla \bar{p} = 0 \tag{3.13}
\]

\[
\bar{u} \cdot \nabla \bar{p} + \gamma \bar{\rho} \nabla \cdot \bar{u} = \gamma \bar{q} \tag{3.14}
\]

Substituting the flow variable decompositions in Eq. 3.9, Eq. 3.10, and Eq. 3.11 neglecting second order terms of the fluctuating components, with the assumption that for low Mach number flows, the effect of the step change in the pressure is negligible compared to the change in the mean velocity or mean density, and the spacial gradient of the mean pressure as well as the mean velocity can be considered small. Using steady state Eq. 3.12, Eq. 3.13 and Eq. 3.14, the governing linear equations for describing the perturbations are obtained as

\[
\bar{\rho} \gamma \frac{\partial u'}{\partial t} + \bar{\rho} \gamma \bar{u} \cdot \nabla u' + \nabla p' = 0 \tag{3.15}
\]

\[
\frac{\partial p'}{\partial t} + \bar{u} \cdot \nabla p' + \gamma \bar{\rho} \nabla \cdot u' = \gamma q' \tag{3.16}
\]
3.1. The Governing Equations

By taking the divergence of Eq. 3.15 and taking \((\frac{\partial}{\partial t} + \bar{u} \cdot \nabla)\) on Eq. 3.16, and after some algebraic manipulation, the following wave equation can be obtained.

\[
\frac{\partial^2 p'}{\partial t^2} - (1 - \bar{u}^2) \nabla^2 p' + 2\bar{u} \frac{\partial (\nabla p')}{\partial t} = \gamma \left( \frac{\partial q'}{\partial t} + \bar{u} \nabla q' \right)
\]  

(3.17)

When the mean flow Mach number is small, the wave equation 3.17 in its non-dimensional form can be simplified as

\[
\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \gamma \left( \frac{\partial q'}{\partial t} \right)
\]  

(3.18)

Since the software used for solving the equations (SYSNOISE) utilizes the dimensional form of equation, dimensionalization is resorted to be consistent with the software implementation. By back substitution of dimensionless quantities in Eq. 3.18, the inhomogeneous wave equation can be obtained as

\[
\frac{1}{c^*^2} \frac{\partial^2 p^{*'}}{\partial t^{*2}} - \nabla^2 p^{*'} = \frac{(\gamma - 1)}{c^*^2} \left( \frac{\partial q^{*'}}{\partial t} \right)
\]  

(3.19)

By denoting acoustic pressure perturbation \(p^{*'} = Pe^{j\omega t}\), where sinusoidal pressure oscillations are assumed with \(\omega\) the angular frequency and \(P\) is the local complex pressure amplitude, also referred to as acoustic pressure, and \(j = \sqrt{-1}\) and \(f = \omega/2\pi\), where \(f\) is the frequency of oscillations. Here acoustic pressure is assumed to be steady (the amplitude of oscillations are not varying with time) and the governing equation for \(P\) can be obtained as the second order Helmholtz equation,

\[
\nabla^2 P + k^2 P = -j \frac{(\gamma - 1)}{c^*^2} \omega Q
\]  

(3.20)

where \(k = \omega/c^*\) is the acoustic wave number.

3.1.1 Boundary conditions

Three types of boundary conditions are in general applied to the domain boundary. Let \(\Omega\) denote the boundary of the domain \(\mathcal{V}\). The part of the boundary, where each type of boundary condition is applied, is denoted as \(\Omega_v\), \(\Omega_p\) and \(\Omega_Z\). The three surfaces together constitute the domain boundary \(\Omega\). The three conditions are

1. Imposed normal velocity: \(v_n = \frac{j}{\rho_0 \omega} \frac{\partial P}{\partial n} = \bar{v}_n\) on \(\Omega_v\)

2. Imposed pressure: \(P = \bar{P}\) on \(\Omega_p\)

3. Imposed normal impedance: \(p = \bar{Z} \cdot v_n = \frac{v_n}{A} = \frac{j}{\rho_0 \omega} \frac{\partial P}{\partial n} = \frac{j}{\rho_0 \omega A} \frac{\partial P}{\partial n}\) on \(\Omega_Z\)
3.1. Method of solution

To solve the above defined acoustic problem, through the use of finite element method, transformation is sought by weighted residual formulation. An approximation is introduced by expressing the pressure field in terms of a set of prescribed shape functions, which are locally defined within small sub-domains (‘finite elements’) of the bounded fluid domain. In this way, the original problem of determining the pressure field at any position in the fluid domain may be approximately transformed into a problem of determining the pressure at some discrete positions in the fluid domain. This transformation results in a set of algebraic equations (‘finite element model’).

A steady state acoustic pressure field in a bounded fluid domain $\mathcal{V}$ as a pressure field, for which the integral equation

$$\int_{\mathcal{V}} \tilde{p} \left( \nabla^2 P + k^2 P + j \frac{(\gamma - 1)}{c^2} \omega Q \right) \, d\mathcal{V} = 0 \quad (3.21)$$

is satisfied for any weighting function $\tilde{p}$, that is bounded and uniquely defined within volume $\mathcal{V}$ and on its boundary surface $\Omega$. The weighted residual formulation may be further transformed as

$$\int_{\mathcal{V}} [\nabla \cdot (\tilde{p} \nabla P)] \, d\mathcal{V} - \int_{\mathcal{V}} [\tilde{p} \cdot \nabla P] \, d\mathcal{V} + \omega^2 \int_{\mathcal{V}} \left( \frac{1}{c^2} \tilde{p} P \right) \, d\mathcal{V} + \int_{\mathcal{V}} j \frac{(\gamma - 1)}{c^2} \omega \tilde{p} Q \, d\mathcal{V} = 0 \quad (3.22)$$

According to the divergence theorem, the integral of the normal component of a vector field taken over a closed surface $\Omega$, is equal to the integral of the divergence of the vector field taken over the volume $\mathcal{V}$. Then, first term in equation 3.22 can be written as

$$\int_{\mathcal{V}} [\nabla \cdot (\tilde{p} \nabla P)] \, d\mathcal{V} = \int_{\Omega} \left( \frac{\partial P}{\partial n} \right) \, d\Omega = \int_{\Omega} (j \rho_0 \omega \tilde{p} v) \, d\Omega \quad (3.23)$$

Thus, the resulting equation yields the ‘weak form’ of the weighted residual formulation of the Helmholtz equation as

$$\int_{\mathcal{V}} [\nabla \cdot (\tilde{p} \nabla P)] \, d\mathcal{V} - \omega^2 \int_{\mathcal{V}} \left( \frac{1}{c^2} \tilde{p} P \right) \, d\mathcal{V} = \int_{\mathcal{V}} j \frac{(\gamma - 1)}{c^2} \omega \tilde{p} Q \, d\mathcal{V} - \int_{\Omega} (j \rho_0 \omega \tilde{p} v) \, d\Omega \quad (3.24)$$

In the finite element method, the fluid domain $\mathcal{V}$ is discretized into a number of small sub-domains $\mathcal{V}_e$ and a number of nodes $n_e$. Within each element, the distribution of the field variable $P$ is approximated as an expansion $\hat{P}$ in terms of prescribed shape functions $N^e_i$. Therefore

$$\hat{P} = \sum_{i=1}^{n_e} N^e_i \hat{P}_i \quad (3.25)$$

The corresponding pressure gradient becomes

$$\nabla P \approx \nabla \hat{P} = [\partial] \{N\} \{\hat{P}\} \equiv [B] \{\hat{P}_i\} \quad (3.26)$$
The first term of Eq. 3.24 yields
\[ \int_\gamma (\nabla \bar{p} \cdot \nabla \bar{p}) d\gamma = \{\bar{p}_i\}^T \left( \int_\gamma [B]^T \cdot [B] d\gamma \right) \{\bar{P}_i\} = \{\bar{P}_i\}^T [K] \{\bar{P}_i\} \] (3.27)

where the matrix \([K]\) is called the acoustic stiffness matrix. The second term of Eq. 3.24 yields
\[ -\omega^2 \int_\gamma \left( \frac{1}{\epsilon_0^2} \bar{p} P \right) d\gamma' = -\omega^2 \{\bar{p}_i\}^T \left( \int_\gamma \frac{1}{\epsilon_0^2} [N]^T \cdot [N] d\gamma' \right) \{\bar{P}_i\} = -\omega^2 \{\bar{p}_i\}^T [M] \{\bar{P}_i\} \] (3.28)

where the matrix \([M]\) is called the acoustic mass matrix. The third term of Eq. 3.24 yields
\[ \int_\gamma j \left( \frac{\gamma - 1}{c^2} \right) \omega \bar{p} Q d\gamma' = \{\bar{P}_i\}^T \left( \int_\gamma j \left( \frac{\gamma - 1}{c^2} \right) \omega [N]^T Q d\gamma' \right) \{\bar{P}_i\} \equiv \{\bar{p}_i\}^T \{Q_i\} \] (3.29)

where \([Q_i]\) is known as the acoustic excitation vector.

The last term in Eq. 3.24 accommodates the introduction of boundary conditions. The integration over the surface may be regarded as a sum of the integrations over the sub-surfaces \(\Omega_s, \Omega_Z\) and \(\Omega_p\). The boundary conditions of normal velocity, normal impedance and pressure must be satisfied on the corresponding surfaces. The last term in Eq. 3.24 may be expressed as
\[ -\int_{\Omega_s} (j \rho_0 \omega \bar{p} \bar{v}) d\Omega - \int_{\Omega_Z} (j \rho_0 \omega \bar{p} \bar{v}) d\Omega - \int_{\Omega_p} (j \rho_0 \omega \bar{p} \bar{v}) d\Omega \] (3.30)

The terms of the above expression can be written in terms of shape functions as
\[ -\int_{\Omega_s} (j \rho_0 \omega \bar{p} \bar{v}) d\Omega = \{\bar{p}\}^T \left( \int_{\Omega_s} -j \rho_0 \omega [N]^T \cdot \bar{v}_n d\Omega \right) \{\bar{P}\} = \{\bar{p}\}^T \{v\} \] (3.31)
\[ -\int_{\Omega_Z} (j \rho_0 \omega \bar{p} \bar{v}) d\Omega = -j \omega \{\bar{p}\}^T \left( \int_{\Omega_Z} \rho_0 \bar{A}[N]^T \cdot [N] d\Omega \right) \{\bar{P}\} = -j \omega \{\bar{p}_i\}^T \{C\} \cdot \{\bar{P}_i\} \] (3.32)
\[ -\int_{\Omega_p} (j \rho_0 \omega \bar{p} \bar{v}) d\Omega = \{\bar{p}\}^T \left( \int_{\Omega_p} (-j \rho_0 \omega [N]^T v \cdot n) \cdot d\Omega \right) \{\bar{P}\} = \{\bar{p}_i\}^T \{p\} \] (3.33)

By substituting Eq. 3.28, Eq. 3.29, Eq. 3.31, Eq. 3.32 and equation 3.33, in Eq. 3.24, the weak form of the weighted residual formulation of the Helmholtz equation, including the boundary conditions becomes
\[ \{\bar{p}_i\}^T (\{K\} + j \omega \{C\} - \omega^2 \{M\}) \cdot \{\bar{P}_i\} = \{\bar{p}\}^T \cdot (\{Q\} + \{v_n\} + \{p\}) \] (3.34)

The final equation to be solved is given by
\[ ([K] + j \omega [C] - \omega^2 [M]) \{p_i\} = (\{Q\} + \{v_n\} + \{p\}) \] (3.35)

Equation 3.35 is the resulting finite element model for a coupled acoustic problem. The above equations are solved through SYSNOISE software.
3.2 SYSNOISE Software

SYSNOISE [64] is a program for modelling acoustics and vibro-acoustics, using the finite element and boundary element methods. It calculates acoustic wave behaviour in fluids, and two-way fluid-structure interaction, using implementations of the finite element and boundary element methods focused on optimal solutions for acoustic problems. The acoustic domain can be closed, open, or partially-open, including homogeneous fluid or multiple fluids. A coupled structure can be wholly or partly connected to fluid.

SYSNOISE predicts the sound wave propagation. It calculates a wide variety of results such as sound pressure and radiated sound power, acoustic velocities and intensities, contributions of panel groups to the sound, energy densities, vibro-acoustic sensitivities, normal modes and structural deflections.

SYSNOISE utilizes numerical methods based on the direct and indirect boundary element method and a pressure formulation for acoustic finite and infinite element modelling. A finite element model represents the elasticity of the fluid loaded structure. Calculations are performed in both frequency and time domains.

SYSNOISE models acoustics and vibro-acoustics as a wave phenomenon. In most cases, the modelling is carried out in the frequency domain, thus using the Helmholtz form of the wave equation.

The Direct Response frequency analysis procedure solves the following system of equations for selected frequencies.

\[
([K] + j\omega[C] - \omega^2[M])\{p\}' = ([Q] + \{u_n\}' + \{p\})
\]  

(3.36)

Right hand side of equation 3.36 is proportional to the normal velocity boundary conditions imposed on the faces of the mesh as well as any forcing function that can be applied in the acoustic domain as defined by the user. The stiffness, damping and mass matrices are computed only once as they are independent of the frequency. At each frequency, the system of equations is set up and solved to obtain the pressure distribution.

Figure 3.1 shows the afterburner acoustic mesh that was modelled for FEM acoustic analysis. The acoustic mesh is modelled from the slave combustor exit onwards. Centre body, symmetrical struts and flame stabiliser are taken into account in meshing. Bypass air column is provided where the perforate liner is positioned.
As the main concern is to capture transverse acoustic pressure variation along the cross section of the afterburner, three dimensional mesh is chosen. For the test section, the first cut on frequency in transverse direction is 847 Hz at acoustic speed of 343 m/s. This number will be even higher for the case of hot slave combustor and afterburner. Thus the frequency of interest is chosen up to 2000 Hz in the analysis. The mesh consists of 266707 TETR4 and 2206 PENT6 elements. Numerical pollution can be avoided by 6 elements per wavelength. The mesh design rule is

\[ \Gamma < \frac{c^*}{6f_{\text{max}} \sqrt{\frac{2\pi f_{\text{max}}}{c^*}}} \]  

The maximum frequency up to which calculations are carried out is 2000 Hz, which requires \( \Gamma \), element length to be 10 mm. The mesh is made still finer by choosing element length to be 5 mm because of which the mesh is valid up to 4000 Hz as the software recommends that the mesh size should be such that it is valid up to double the frequency of interest.

In order to predict the linear response of the system subjected to harmonic excitation, steady state dynamic analysis is performed. The panel vibration boundary condition is imposed to excite the acoustic field. Particle velocity along elements at inlet face edge is applied, which is considered as an open end, because the face is connected to a large diverging combustor inlet. With the forcing function, frequency by frequency computation of complex acoustic potentials at each node in the mesh is carried out. By defining the panel vibration as having a complex value of \( 1 + j0 \) m/s for the frequency range of 10 Hz to 2000 Hz, the phase response is relative to the expansion of the source into the acoustic field.

Figure 3.2 gives the normalized acoustic pressure and phase for uncoupled case which is obtained for the given test section by inlet excitation and nozzle admittance boundary condition. Calculations involving perforate liner impedance and nozzle admittance are presented in section 3.3 and section 3.4 respectively.

### 3.3 Acoustic impedance of perforate liner geometry

In the afterburner test section the perforated liner is provided to attenuate the amplitudes of screech frequencies. A few of the perforate liner models are reviewed \[65,66,67\]. The acoustic impedance, \( \bar{Z} \), of perforate liner is a complex function of several physical variables, namely, porosity (assumed to be uniform), mean-flow velocity through the holes or grazing the holes, diameter and thickness. \( \bar{Z} \) is calculated by using the Mechels formula \[63\] which is widely used. This formula is relevant when the perforate thickness is not too large (thickness << 2d). It can be used in the linear range. The non-linearity of the real component of \( \bar{Z} \) appears when \( u'_{n} \geq 3 \sqrt{\frac{\omega \mu}{\rho}} \) where \( u'_{n} \) is the particle velocity in the perforate orifice.

\[ \bar{Z} = \frac{p'}{u'_{n}} = R_{p} + jX_{p} \]  

(3.38)
3.3. Acoustic impedance of perforate liner geometry

Figure 3.2: Uncoupled normalized acoustic pressure and phase

where, resistance of perforate, \( R_p \), is given by

\[
R_p = \frac{1}{\Phi} \sqrt{8\omega \mu \rho^*} \left( 1 + \frac{\text{thk}}{d} \right)
\]  

(3.39)

and reactance of perforate, \( X_p \) is given by

\[
X_p = \frac{1}{\Phi} \omega \rho^* (\text{thk} + 2\Delta(\text{thk})
\]  

(3.40)

and

\[
\Delta(\text{thk}) = 0.85 \frac{d}{2} \left( 1 - 2.34 \frac{d}{2l} \right) \quad \text{for } 0 < \frac{d}{2l} < 0.25
\]  

(3.41)

The frequency dependent boundary condition for the acoustic impedance of the perforate, \( \bar{Z} \), can be imposed in the software by building a table with the values of the functions \([R_p(f), X_p(f)]\) which will be imported as transfer relation boundary conditions as follows,

\[
\begin{bmatrix}
  u'_{n1} \\
  u'_{n2}
\end{bmatrix} = \begin{bmatrix}
  \alpha_1 & \alpha_2 \\
  \alpha_4 & \alpha_5
\end{bmatrix} \begin{bmatrix}
  p'_{1} \\
  p'_{2}
\end{bmatrix} + \begin{bmatrix}
  \alpha_3 \\
  \alpha_6
\end{bmatrix}
\]  

(3.42)

where, \( \alpha_1, \alpha_2, \alpha_4, \alpha_5 \) are the complex admittance coefficients pertaining to porosity and \( \alpha_3, \alpha_6 \) are the complex source coefficients and

\[
\begin{bmatrix}
  \alpha_1 & \alpha_2 \\
  \alpha_4 & \alpha_6
\end{bmatrix} = \begin{bmatrix}
  \frac{1}{z} & -\frac{1}{Dz} \\
  -\frac{1}{Dz} & \frac{1}{Z}
\end{bmatrix}
\]  

(3.43)

where, \( D \) is a factor that takes into account the thickness of the perforate.
3.4 Nozzle admittance — for choked convergent nozzle

Figure 3.3 shows the variation of resistance, $R_p$, with frequency ranging from 10 Hz to 2000 Hz. Figure 3.4 shows the variation of reactance, $X_p$, with the frequency. Three different screech liner configurations are studied by varying porosity from 1.5 % to 4 %. A liner with porosity of 2.5 % is chosen after parametric study. These are implemented in the software as transfer relation boundary condition for the perforate liner.

3.4 Nozzle admittance — for choked convergent nozzle

The behaviour of convergent nozzle for non-isentropic and non-isothermal conditions in the range of oscillatory frequencies is given by the complex admittance for a choked condition. The derivation is adapted from Crocco [68] and presented in Appendix A. Figures 3.5 and 3.6, show the real part and the imaginary part respectively of nozzle admittance ratio for 10 to 2000 Hz.

The equations have been derived making the assumption that the velocity increases linearly with the distance along the nozzle for high frequencies. In this limit, the real part of admittance is independent of frequency and has a value of unity. If we take the general case applicable to any frequency, then the real part of the admittance coefficient varies from $(\gamma - 1)/2$ at low frequency to 1 as frequency tends to $\infty$. To this extent the results in the low frequency zone introduces error in the computation of acoustic field. This aspect is ignored in this analysis, as the main concern of the analysis is for high frequency range (Screech).
3.4. Nozzle admittance — for choked convergent nozzle

Figure 3.4: Reactance ($X_p$) variation with frequency for the perforate liner

Figure 3.5: Real part of nozzle admittance variation with frequency
3.5 Concluding Remarks

The most important aspect related to combustion instability in afterburners is the development of equations and solution of pressure wave-flame interaction. In this work combustion flame response is developed from fundamentals which are unique compared to the body of literature on simple conventional flame response to pressure disturbances of different amplitudes characterized by different length scales. In chapter 4 complete theoretical formulation is presented highlighting the new feature compared to the previous work. The same is implemented as a user defined routine source function in the SYSNOISE software.
Chapter 4

Combustion Source Response Function

4.1 Introduction

In this chapter, the development of theory for pressure wave – flame interaction is presented. The present combustion model is unique compared to the body of literature on simple conventional flame response to pressure disturbances of different amplitudes characterized by different length scales. The previous work in this regard is discussed to a certain extent to highlight the novelty of the newly developed combustion source response function in this thesis.

The combustion in the afterburner can be assumed to be taking place in the premixed mode, as the fuel and air are largely mixed by the time they reach the flame holder, and the flame is stabilized behind the flame holder and the reaction occurs homogeneously over a volume of uniform cross section. The flame is turbulent in nature. The incoming flow has Reynolds number greater than $10^4$. Acoustic wave – flame interactions involve unsteady kinetic, fluid mechanic and acoustic processes over a large range of time scales. Three types of flow disturbances exist such as: vortical, entropy, and acoustic. In a homogeneous, uniform flow, these three disturbance modes propagate independently in the linear approximation. Unsteady heat release also generates entropy and vorticity disturbances. When flow is not accelerated as in the case of uniform area duct, vortical and entropy disturbances can be treated as insignificant, as these disturbances are convected out into atmosphere like an open-ended tube, but these are to be considered in the nozzle.

In some of the earlier papers [69,70,71] very low amplitude acoustic coupling of flames was investigated where the disturbances were of medium wavelength. The assumption of low amplitude of the disturbance meant that the only coupling was through the velocity perturbation. It was then found that the velocity perturbation was only affected by the flame when heat loss came into effect. In the absence of heat loss, such low level acoustics simply regarded the flame as a contact discontinuity swept along with the small unsteady acoustic waves. Under the assumption of small amplitude (of the order of deflagration Mach number), there was no coupling of pressure changes with the temperature disturbances at
the flame. However the situation changes when the amplitude of the disturbance is not small, which is the case with afterburner.

van Harten et al [43] considered acoustic coupling of flames with very long wavelengths (up to 400 Hz) and medium wave lengths (600–3000 Hz). Density effects were included in the first case but not in the second. McIntosh [41] studied the acoustic coupling for medium wavelengths with the restriction of constant density lifted, and replacing with the more realistic assumption that density times thermal conductivity is a constant. In this work, $D\rho^*/k^*/\rho^*/c^*_p$ are assumed to be independent of temperature and proportional to pressure. This assumption has been made from the observation that for most gases the thermal conductivity and $D\rho^*$ vary approximately as $T^{3/4}$, while they are independent of pressure when the gas temperature is sufficiently above the critical temperature [72].

In the above references the flame itself is considered as a discontinuity and the dynamics of concentration and temperature fluctuations within the flame were not accounted for. Hence a formulation, in which the integration is performed across the flame where in the reaction rate expression is formally included in the analysis, is proposed in in this chapter. This approach accounts for the fluctuations of concentration and temperature within the reaction zone. How the flame responds to incoming disturbances is dealt to certain length in the following discussion, to justify the approximations assumed in the development of flame response function. The global heat release rate of the flame is given by equation

$$q^*(t) = \int_{\Omega} \rho^* S \nabla h_R d\Omega_{FL}$$  \hspace{1cm} (4.1)

where the integral is performed over the flame surface area $\Omega_{FL}$ and $\nabla h_R$ is the heat release per unit mass of reactant. The above equation shows the four fundamentally different ways of generating heat release disturbances in a premixed flame: fluctuations in density, flame speed, heat of reaction, or flame surface area.

Fluctuations in the mass flow rate of reactive mixtures into the flame, corresponds to $\rho^* S^*$. The density fluctuations could be caused by both acoustic and entropy fluctuations. The flames burning rate $S$ is sensitive to the perturbations in pressure, temperature, strain rate, or mixture composition that accompany the acoustic wave. The pressure and temperature fluctuations are usually generated by acoustic disturbances, where as strain rate fluctuations are associated with acoustic or vortical velocity fluctuations. Lieuwen [73] has shown that, though the acoustic wave amplification caused by this mechanism is nonzero, it is of the order laminar flame speed Mach number (typically 0.001), therefore quite weak.

The second important aspect is the flame-area disturbances arising from flow velocity perturbations. Elaborate mathematical treatment is available in reference [73]. The flame area acts as a low-pass filter to flow disturbances, so that the amplitudes decay with frequency. The coupling effect is very strong in the low frequency range up to 400 Hz, but decays to zero level through out high frequency range. The response function is primarily dependent on the dimensionless quantity, Strouhal number, defined as $\omega L/u_0$, and it has been shown that as Strouhal number increases, the flame area fluctuations decrease. The results show that beyond Strouhal number of about 10 (corresponding to a frequency of about
4.2 Governing Equations

The combustion in the afterburner can be described using a turbulent flame propagation model. One of the models proposed for turbulent flame propagation is the flame surface density model, in which the turbulent reaction rate is calculated as

\[ \dot{\omega}_f = \dot{\Omega}_f \sigma \]  

(4.2)

where \( \sigma \) is the flame surface area per unit volume, \( \dot{\Omega} \) is the integrated reaction rate over the flame, defined as

\[ \dot{\Omega} = \int_{-\infty}^{+\infty} \dot{\omega}_f dz \]  

(4.3)

\[ = \rho_u Y_{fu} S_L \]  

(4.4)

In the above equation, \( \rho_u \) is the density of the unburnt gas, \( Y_{fu} \) is the mass fraction of the fuel in the unburnt gases, and \( S_L \) the strained laminar flame speed. In the turbulent flame calculations, the strained laminar flame speed is calculated by the unstrained laminar burning velocity multiplied by a correction faction factor for the Karlowitz number, which is estimated based on the turbulence parameters. Assuming that Karlowitz number and the other parameters, such as the flame surface density, are unaffected by the acoustic oscillations in the combustion chamber, the model can be extended to the derivation of combustion source for the afterburner stability analysis. The assumption implicit in this type of analysis is that the turbulence time and space scales are much smaller than the corresponding scales of acoustic oscillations. One-dimensional conservation equations will be used for obtaining the response factor for the heat release rate.

4.3 Flame Description

The flame response is derived from, the one-dimensional conservation of mass, energy and species conservation equations as given below. In a typical afterburner configuration, the length scale of pressure oscillations for transverse modes would be of the order of magnitude of diameter of the afterburner (typically a meter) and the flame thickness would be about a millimeter. The space coordinate is
4.3. Flame Description

normal to the local flame front.

\[ \frac{\partial \rho^*}{\partial t^*} + \frac{\partial \rho^* u^*}{\partial z} = 0 \quad (4.5) \]

\[ \rho^* \frac{\partial Y_i}{\partial t^*} + \rho^* u^* \frac{\partial Y_i}{\partial z} = \frac{\partial}{\partial z} \left( D \rho^* \frac{\partial Y_i}{\partial z} \right) + \dot{w}_i \quad (4.6) \]

\[ \rho^* c_p^* \frac{\partial T^*}{\partial t^*} - \frac{\partial p^*}{\partial t^*} + \rho^* u^* c_p^* \frac{\partial T^*}{\partial z} = \frac{\partial}{\partial z} \left( k^* \frac{\partial T^*}{\partial z} \right) - H_c \dot{w}_f \quad (4.7) \]

The space variable \( z \) is transformed as

\[ d\psi = \rho^* dz, \quad t = t^* \quad (4.8) \]

so that

\[ \frac{\partial t}{\partial t^*} = 1 \quad (4.9) \]

\[ \frac{\partial t}{\partial z} = 0 \quad (4.10) \]

\[ \frac{\partial \psi}{\partial t^*} = - \int_{-\infty}^{z} \frac{\partial \rho^*}{\partial t} dz \quad (4.11) \]

\[ \frac{\partial \psi}{\partial z} = \rho_{-\infty}^* u_{-\infty}^* - \rho^* u^* \quad (4.12) \]

\[ \frac{\partial \psi}{\partial z} = \rho^* \quad (4.13) \]

In Eq. 4.12, the term \( \rho_{-\infty}^* u_{-\infty}^* \) can be chosen as \( \rho_u^* S_u \), where \( S_u \) is the laminar burning velocity corresponding to the steady state conditions and \( \rho_u^* \) the density of the unburnt gases, so that the time derivative term cancels from the conservation equations in steady state. The transformation relationships can be obtained as

\[ \frac{\partial}{\partial t^*} = \frac{\partial}{\partial t} + \frac{\partial \psi}{\partial t} \frac{\partial}{\partial \psi} \quad (4.14) \]

\[ \frac{\partial}{\partial z} = \rho^* \frac{\partial}{\partial \psi} \quad (4.15) \]

The transformed conservations can be written as

\[ \frac{\partial Y_i}{\partial t^*} + \rho_u^* S_u \frac{\partial Y_i}{\partial \psi} = \frac{\partial}{\partial \psi} \left( D \rho^* \frac{\partial Y_i}{\partial \psi} \right) + \frac{\dot{w}_i}{\rho^*} \quad (4.17) \]

\[ \frac{\partial T^*}{\partial t^*} - \frac{1}{\rho^* c_p^*} \frac{\partial p^*}{\partial t^*} + \rho_u^* S_u \frac{\partial T^*}{\partial \psi} = \frac{\partial}{\partial \psi} \left( k^* \rho^* \frac{\partial T^*}{c_p^*} \frac{\partial}{\partial \psi} \right) - H_c \frac{\dot{w}_f}{\rho^* c_p^*} \quad (4.18) \]

Note that \( \partial p^*/\partial \psi \) is assumed zero here. At this stage unity Lewis number \([74,41]\) is assumed and \( \rho^* \) and \( k^* \rho^*/c_p^* \) being independent of temperature and proportional to pressure. This assumption has been made from the observation that for most gases the thermal conductivity and \( D \rho \) varies approximately as \( T^{3/4}, \)
while they are independent of pressure when the gas temperature is sufficiently above the critical temperature \cite{72} The flame thickness, being small compared to the wave length of the acoustic oscillations, is considered to be a function of time but not a function of space. The following substitutions are made in the conservation equations.

\[
\theta = \left(\frac{\bar{\rho}_-\infty S_u}{k^* \rho^* / c_p^*}\right)^2 t 
\]

\[
\xi = \frac{\bar{\rho}_-\infty S_u}{k^* \rho^* / c_p^*} \psi \quad (4.20)
\]

\[
\tau = \frac{T^* - T^-\infty}{T^-\infty - T^*\infty} \quad (4.21)
\]

The conservation equation equations become

\[
\frac{\partial Y_i}{\partial \theta} + \frac{\rho_u}{\bar{\rho}_-\infty} \frac{\partial Y_i}{\partial \xi} = p \frac{\partial^2 Y_i}{\partial \xi^2} + \dot{\omega}_i \quad (4.22)
\]

\[
\frac{\partial \tau}{\partial \theta} - \frac{1}{\rho^* c_p^*(T^*_\infty - T^-\infty)} \frac{\partial p}{\partial \theta} + \frac{\rho_u}{\bar{\rho}_-\infty} \frac{\partial \tau}{\partial \xi} = p \frac{\partial^2 \tau}{\partial \xi^2} - \frac{H_c}{c_p^*(T^*_\infty - T^-\infty)} \dot{\omega}_f \quad (4.23)
\]

where

\[
\dot{\omega}_i = \left(\frac{k^* \rho^* / c_p^*}{\bar{\rho}_-\infty S_u}\right) \left(\frac{\dot{\omega}_i}{\rho^*}\right) \quad (4.24)
\]

Two conserved scalars $\beta_1$ and $\beta_2$ are defined as

\[
\beta_1 = Y_f - \frac{Y_o}{\nu} \quad (4.25)
\]

\[
\beta_2 = \tau + \frac{H_c Y_f}{c_p^*(T^*_\infty - T^-\infty)} \quad (4.26)
\]

where $\nu$ is the stoichiometric air to fuel ratio. At steady conditions, the solutions are obtained as

\[
\bar{\beta}_1 = Y_{f,-\infty} - \frac{Y_{o,-\infty}}{\nu} \quad (4.27)
\]

\[
\bar{\beta}_2 = \frac{H_c Y_{f,-\infty}}{c_p^*(T^*_\infty - T^-\infty)} \quad (4.28)
\]

Assuming that after reaction in the flame the fuel mass fraction goes to zero, applying condition at $\xi = \infty$ leads to the result

\[
\frac{H_c}{c_p^*(T^*_\infty - T^-\infty)} = \frac{1}{Y_{f,-\infty}} \quad (4.29)
\]

It is assumed that the liquid injection rate is insensitive to the acoustic pressure oscillations and hence the unburnt mass fraction can be considered to be constant. Since the diffusion rate of fuel and oxidizer are equal, the response of $\beta_1$ to pressure oscillations can be considered to be zero. The equation for $\beta_2$ can be written as

\[
\frac{\partial \beta_2}{\partial \theta} - \frac{1}{\rho^* c_p^*(T^*_\infty - T^-\infty)} \frac{\partial p}{\partial \theta} = p \frac{\partial^2 \beta_2}{\partial \xi^2} - \frac{\dot{\omega}_f}{Y_{f,-\infty}} \quad (4.30)
\]
4.3. Flame Description

The second term of Eq. 4.30 can be shown to be equal to \( \partial \beta_2 / \partial \theta \) under the assumption of spatially invariant pressure across the flame and isentropic conditions at the cold boundary. Hence Eq. 4.30 can be rewritten as

\[
\frac{\partial (\beta_2 - \beta_2, -\infty)}{\partial \theta} + \frac{p_u}{\bar{p}, -\infty} \frac{\partial (\beta_2 - \beta_2, -\infty)}{\partial \xi} = \frac{p \partial^2 (\beta_2 - \beta_2, -\infty)}{\bar{p}} - \frac{\dot{\omega}_f}{Y_{f, -\infty}} \tag{4.31}
\]

The only solution to Eq. 4.31 which is finite at both \( \xi = \pm \infty \) is \( \beta_2 = \beta_2, -\infty \). These relations can now be utilized to represent all the dependent variables as function of a single dependent variable. \( \tau \) is chosen as the dependent variable to be solved. Hence

\[
\frac{Y_f}{Y_{f, -\infty}} = 1 - (\tau - \tau_{-\infty}) \tag{4.32}
\]

\[
\frac{Y_o}{Y_{o, -\infty}} = 1 - \phi(\tau - \tau_{-\infty}) \tag{4.33}
\]

where \( \phi \) is the equivalence ratio \( \equiv Y_{f, -\infty}/Y_{o, -\infty} \). Hence the equation to be solved can be written as

\[
\frac{\partial (\tau - \tau_{-\infty})}{\partial \theta} + \frac{p_u}{\bar{p}, -\infty} \frac{\partial (\tau - \tau_{-\infty})}{\partial \xi} = \frac{p \partial^2 (\tau - \tau_{-\infty})}{\bar{p}} - \frac{\dot{\omega}_f}{Y_{f, -\infty}} \tag{4.34}
\]

### 4.3.1 The reaction rate

The reaction rate is expressed as that of a second order single step reaction. The form chosen is

\[
\dot{\omega}_f = -AM_f e^{-E/RT^*} T^* c_f c_o \tag{4.35}
\]

\[
= -\frac{A}{M_o} e^{-E/RT^*} \rho^2 T^* Y_f Y_o \tag{4.36}
\]

\[
= -\left( \frac{AM}{M_o R} \right) e^{-E/RT^*} \rho^* \rho^* Y_f Y_o \tag{4.37}
\]

The reaction rate can be expressed in terms of a single variable \( \tau \) as

\[
-\frac{\dot{\omega}}{Y_{f, -\infty}} = \left( \frac{k^* \rho^*/c_p^*}{\rho_u S_u} \right) \left( \frac{AM Y_{o, -\infty} \bar{p}}{M_o R} \right) e^{-E/RT^*} (1 - \beta(1 - \tau_{-\infty}) - (\tau - \tau_{-\infty})) \]

\[
\times \left( \frac{\bar{p}}{\bar{p}} \right) [1 - (\tau - \tau_{-\infty})] [1 - \phi(\tau - \tau_{-\infty})] \tag{4.38}
\]

where

\[
\beta \equiv \frac{T^*_{x, -\infty} - T^*_{x, \infty}}{T^*_\infty} \tag{4.39}
\]

### 4.3.2 Steady state solution

At steady state

\[
\tau_{-\infty} = 0 \tag{4.40}
\]

\[
\frac{\bar{p}}{p} = 1 \tag{4.41}
\]
and putting the time derivative to zero the conservation equation becomes

\[ \frac{d\tau}{d\xi} = \frac{d^2\tau}{d\xi^2} + \lambda e^{-\Theta/[1-\beta(1-\tau)]}(1-\tau)(1-\tau) \]

(4.42)

where

\[ \lambda \equiv \left( \frac{k^*\rho^*}{c_p^*} \right) \left( \frac{AMY_{\infty}}{M_oR} \right) \]  

(4.43)

\[ \Theta \equiv \frac{E}{RT^*_\infty} \]  

(4.44)

The steady state solution is obtained by the technique of matched asymptotic expansion, with the assumption of large activation (i.e. large \( \Theta \)). Under this assumption an outer layer, where balance of conduction and convection takes place and an inner layer where balance of reaction and conduction takes place. Substituting

\[ \chi \equiv \frac{d\tau}{d\xi} \]  

(4.45)

Eq. 4.42 becomes

\[ \chi - \chi \frac{d\chi}{d\tau} = \lambda e^{-\Theta/[1-\beta(1-\tau)]}(1-\tau)(1-\tau) \]

(4.46)

Under the assumption of large \( \Theta \), the order of magnitude of right hand side is lower than any powers of \( 1/\Theta \) and hence the outer solution is obtained as

\[ \chi = \tau \]

(4.47)

Since this solution is obviously not satisfactory uniformly everywhere in the domain, an inner equation is obtained in the region \( \tau \rightarrow 1 \) A new variable \( \eta \) is defined as

\[ \eta \equiv \Theta(1-\tau) \]

(4.48)

and Eq. 4.46 becomes

\[ \chi + \Theta \chi \frac{d\chi}{d\eta} = \Lambda e^{-\beta\eta} \left[ \frac{1}{\Theta} \right] \]

(4.49)

where

\[ \Lambda \equiv \lambda e^{-\Theta} \]

(4.50)

An approximation of \((1-\tau)\) is small has been made in deriving Eq. 4.49. The analysis varies slightly between the case when \( \phi \sim 1 \) and \( \phi << 1 \). The classical solution using matched asymptotic expansion is reproduced here for the cases of lean mixture and for mixture close to stoichiometric conditions.
4.3. Flame Description

Analysis for $\phi \ll 1$

When $\phi$ is not very close to unity, the following expansion can be made for the variables in Eq. 4.49.

\[
\chi = \chi_0 + \frac{\chi_1}{\Theta} + \frac{\chi_2}{\Theta^2} + \cdots
\]  
\[
\Lambda = \Theta^2 \left\{ \Lambda_0 + \frac{\Lambda_1}{\Theta} + \frac{\Lambda_2}{\Theta^2} + \cdots \right\}
\]

and matching the coefficients of equal powers of $\Theta$ the following equations are obtained.

\[
\chi_0 \frac{d\chi_0}{d\eta} = (1 - \phi)\Lambda_0 \eta e^{-\beta \eta}
\]  \hspace{1cm} (4.53)

\[
\chi_0 + \chi_0 \frac{d\chi_1}{d\eta} + \chi_1 \frac{d\chi_0}{d\eta} = (1 - \phi)\Lambda_1 \eta e^{-\beta \eta} - \phi \Lambda_0 \eta^2 e^{-\beta \eta}
\]  \hspace{1cm} (4.54)

Integration of Eq. 4.53 and applying the boundary condition at $\eta = 0$ ($\tau = 1$ or $\xi \to \infty$)

\[
\frac{\chi_0^2}{2} = \frac{(1 - \phi)\Lambda_0}{\beta^2} \{1 - (1 + \beta \eta)e^{-\beta \eta}\}
\]  \hspace{1cm} (4.55)

Matching the inner and the outer solutions using Kaplun’s matching theorem [75], which states

\[
\left( \text{Inner limit of the outer solution} \right) = \left( \text{Outer limit of the inner solution} \right)
\]  \hspace{1cm} (4.56)

we get

\[
\frac{(1 - \phi)\Lambda_0}{\beta^2} = \frac{1}{2}
\]  \hspace{1cm} (4.57)

The solution, which is uniformly valid for the entire region is obtained as

\[
\chi = \tau + \left\{ 1 - \left[ 1 + \beta \Theta (1 - \tau) \right] e^{-\beta \Theta (1 - \tau)} \right\}^{1/2} - 1
\]  \hspace{1cm} (4.58)

The burning rate can be obtained by substituting the variables of Eq. 4.57 as

\[
(\rho_u S_u)^2 = 2 \left( \frac{1 - \phi}{\beta^2} \right) \left( \frac{k^* \rho^*}{\epsilon^*_p} \right) \left( \frac{A M Y_{\alpha, -\infty} \bar{p}}{M_o R} \right) \frac{e^{-E/RT_\infty}}{(E/RT_\infty)^2}
\]  \hspace{1cm} (4.59)

Analysis for $\phi \sim 1$

When $\phi \sim 1$, we define $\alpha \equiv \Theta (1 - \phi)$ which is $O(1)$. Hence Eq. 4.49 can be written as

\[
\chi + \Theta \chi \frac{d\chi}{d\eta} = \Lambda e^{-\beta \eta} \frac{\eta}{\Theta^2} (\alpha - \phi \eta)
\]  \hspace{1cm} (4.60)

The expansion for $\Lambda$ is changed slightly to

\[
\chi = \chi_0 + \frac{\chi_1}{\Theta} + \frac{\chi_2}{\Theta^2} + \cdots
\]  \hspace{1cm} (4.61)

\[
\Lambda = \Theta^3 \left\{ \Lambda_0 + \frac{\Lambda_1}{\Theta} + \frac{\Lambda_2}{\Theta^2} + \cdots \right\}
\]  \hspace{1cm} (4.62)
in order to match the terms. Matching the coefficients of equal powers of $\Theta$ gives

$$\chi_0 \frac{d\chi_0}{d\eta} = \Lambda_0 e^{-\beta \eta}(\alpha - \phi \eta)$$  \hspace{1cm} (4.63)

$$\chi_0 + \chi_0 \frac{d\chi_1}{d\eta} + \chi_1 \frac{d\chi_0}{d\eta} = \Lambda_1 e^{-\beta \eta}(\alpha - \phi \eta)$$  \hspace{1cm} (4.64)

The solution of Eq. 4.63 with the boundary condition applied at $\tau = 1$ is

$$\frac{\chi_0^2}{2} = \Lambda_0 \frac{2\phi + \alpha \beta}{\beta^3} \left\{ 1 - \left( \frac{\beta^2 \phi}{2\phi + \alpha \beta} \eta^2 + \beta \eta + 1 \right)^{e^{\beta \eta}} \right\}$$  \hspace{1cm} (4.65)

and application of matching condition gives

$$\Lambda_0 \frac{2\phi + \alpha \beta}{\beta^3} = \frac{1}{2}$$  \hspace{1cm} (4.66)

The solution uniformly valid is

$$\chi = \tau - 1 + \left\{ 1 - \left( \frac{\phi}{2\phi + \Theta \beta(1 - \phi)} \right) e^{\Theta \beta(1 - \tau)} \right\}^{1/2}$$  \hspace{1cm} (4.67)

The burning rate is obtained as

$$(\rho_u S_u)^2 = 2 \left( \frac{2\phi + \Theta \beta(1 - \phi)}{\Theta^3 \beta^3} \right) \left( \frac{k^* \rho^*}{c_p} \right) \left( \frac{AMY_{\infty} \infty}{M_o R^2} \right) e^{-E/RT_{\infty}}$$  \hspace{1cm} (4.68)

### 4.4 Unsteady Response

The following substitutions are made into Eq. 4.34

$$\tau_g \equiv \tau - \tau_{\infty}$$  \hspace{1cm} (4.69)

$$\frac{p}{\bar{p}} = 1 + p'$$  \hspace{1cm} (4.70)

and obtain

$$\frac{\partial \tau_g}{\partial \theta} + \frac{\partial \tau_g}{\partial \xi} = (1 + p') \frac{\partial^2 \tau_g}{\partial \xi^2}$$

$$+ \lambda e^{-\Theta/\{1 - \beta(1 - \tau_g) + \beta \tau_{\infty}\}}(1 + p')(1 - \tau_g)(1 - \phi \tau_g)$$  \hspace{1cm} (4.71)

The linearized response equation can be derived from this as

$$\frac{\partial \tau_g'}{\partial \theta} + \frac{\partial \tau_g'}{\partial \xi} = \frac{\partial^2 \tau_g'}{\partial \xi^2} + p' \frac{\partial^2 \tau}{\partial \xi^2} + \omega'$$  \hspace{1cm} (4.72)

where

$$\omega' = \tilde{\omega} \left\{ \frac{\Theta \beta}{[1 - \beta(1 - \tau)]^2} (\tau_{\infty} - \tau_g') + p' - \left( \frac{1}{1 - \tau} + \frac{\phi}{1 - \phi \tau} \right) \tau_g' \right\}$$  \hspace{1cm} (4.73)
4.4. Unsteady Response

Assumption of isentropic condition in the unburnt gases imply

\[ \tau_{-\infty} = \frac{(1 - \beta)(\gamma - 1) p'}{\gamma} \]  

and hence

\[ \dot{\omega}' = \ddot{\omega} \left\{ \frac{\Theta(1 - \beta)(\gamma - 1)}{\gamma[1 - \beta(1 - \tau)]^2 + 1} \right\} p' + \right. \]

\[ \left. \ddot{\omega} \left\{ \frac{\Theta \beta}{[1 - \beta(1 - \tau)]^2} - \frac{1}{1 - \tau} - \frac{\phi}{1 - \phi \tau} \right\} \right\} \tau_g' \]  

Substituting

\[ \tau_g' = T_g e^{3\omega \theta} \]  

\[ p' = P e^{3\omega \theta} \]

we get

\[ j\omega T_g + \frac{dT_g}{d\xi} = \frac{d^2T_g}{d\xi^2} + \frac{d^2\tau}{d\xi^2} + f_1(\tau)P + f_2(\tau)T_g \]  

where

\[ f_1(\tau) = \ddot{\omega} \left\{ \frac{\Theta(1 - \beta)(\gamma - 1)}{\gamma[1 - \beta(1 - \tau)]^2 + 1} \right\} \]

\[ f_2(\tau) = \ddot{\omega} \left\{ \frac{\Theta \beta}{[1 - \beta(1 - \tau)]^2} - \frac{1}{1 - \tau} - \frac{\phi}{1 - \phi \tau} \right\} \]

Dividing Eq. 4.78 by \( P \) and defining \( T_g \equiv T_g/P \)

\[ j\omega T_g + \frac{dT_g}{d\xi} = \frac{d^2T_g}{d\xi^2} + \frac{d^2\tau}{d\xi^2} + f_1(\tau) + f_2(\tau)T_g \]  

The response of heat release rate to pressure oscillations is given by

\[ Q = \int_{-\infty}^{\infty} \{ f_1(\tau) + f_2(\tau)T_g \} d\xi \]  

Integration of Eq. 4.78 gives the result

\[ \omega \int_{-\infty}^{\infty} T_g d\xi = Q \]  

The independent variable of Eq. 4.81 is transformed to \( \bar{\tau} \) and we get

\[ j\omega T_g + \chi \frac{dT_g}{d\bar{\tau}} = \chi \frac{d^2T_g}{d\bar{\tau}^2} + \chi \frac{d^2\tau}{d\bar{\tau}^2} + f_1(\bar{\tau}) + f_2(\bar{\tau})T_g \]  

Equation 4.84 is a linear equation in \( T_g \), which can be solved with the boundary condition \( T_g = 0 \) at \( \bar{\tau} = 0 \) and \( \bar{\tau} = 1 \), and the response function \( Q \) can be obtained from equation 4.82 or equation 4.83. The variable, \( \chi \), can be obtained
4.4. Unsteady Response

Figure 4.1: Normalized heat release rate response (Amplitude) to acoustic pressure perturbations compared with McIntosh

Figure 4.2: Normalized heat release rate response (Phase) to acoustic pressure perturbations compared with McIntosh
from equation 4.58 or equation 4.67 depending on the range of equivalence ratio. Only numerical integration is attempted in the present case.

Figures 4.1 and 4.2, show the amplitude and phase of heat release response to acoustic fluctuations for a frequency range of 0 to 2000 Hz. The results obtained by McIntosh [41] is also included for comparison. As can be seen, the real part of the response function increases to a constant value as and the phase increases from $-90^\circ$ to a positive maximum and decreases to zero. This behaviour is inbuilt in the one dimensional flame description. The reaction rate considered is of second order, which implies that the rate increases as the square of pressure. However as the reaction rate increases there will be a corresponding reduction in the reactant concentration, which in turn reduces the reaction rate. The response of reactant concentration is slower compared to the response of the reaction rate itself because some time is required for the concentration to come down to the equilibrium (or quasi-steady) values. Hence at high frequencies the reaction rate responds to the changes in pressure without any phase lag. However, the results of McIntosh seem to show unrestricted increase in the amplitude ratio as frequency increases. As indicated in Section 4.1 the dependence of concentration and temperature fluctuations within the reaction zone was not accounted in Ref. [41] and hence the unrestricted linear growth of FRF, which is not physically realistic.

In the above analysis the effects due to the changes in mixture ratio, the flame surface density and the surface area of the flame brush have been neglected. The earlier authors have considered these in their analysis, while neglecting the effects of flame response itself. The primary consideration has been the flame surface area variation. Much of these effects are likely to be present at low frequencies and low equivalence ratios. Since screech analysis in afterburners, with which we are concerned in this thesis, corresponds to equivalence ratio close to one and relatively high frequencies, the current model proposed is expected to be more appropriate than the older combustion response models.

Unlike the response to the velocity fluctuations, the response function due to pressure fluctuations increase with frequency as shown in Fig. 4.1. The magnitude of response function due to pressure fluctuations is much larger than that due to velocity fluctuations even at frequencies as low as 100 Hz.

### 4.5 Solution Procedure and software implementation

Refer to section 3.2 for general solution procedure. In this section the implementation of user defined combustion source response function in SYSNOISE is presented.

In the experiments, though within the pre-heater non-uniform temperature distribution exists, the same is properly nullified by its construction itself, by the time the vitiated products reach the entrance of the afterburner. This aspect is closely examined in the testing by having a temperature rake at the entry to afterburner with six points, as the temperature determines its acoustic propagation speed and in turn the characteristic impedance of a fluid. Thus uniform temperature profile is made available within the test section. Based on the temperature and on the mean pre heater pressure, acoustic speed and fluid density, the ma-
terial is defined for the entire acoustic domain. The complex convergent nozzle admittance which is computed separately for the choked condition is applied at the entry to the nozzle. In the first run the acoustic pressure distribution in the entire region of acoustic mesh is calculated.

Further, the acoustics in the afterburner are forced by the unsteady heat release rate. To this effect, the gain from the unsteady heat release rate to panel vibration velocity is obtained by the equations given in section 4.2. The combustion response function which is obtained by solving equation 4.82 is imposed in the flame stabilizer region of acoustic mesh.

The coupled combustion oscillations—amplitude and phase—due to reheat operation were estimated by superimposing heat release fluctuations on the acoustic pressure history obtained by solving the homogeneous wave equation (refer Fig 3.2). Figure 4.3 shows coupled normalized acoustic pressure amplitude and phase spectrum.

Then, admittance—that pertains to screech liner—boundary condition through transfer relation is imposed on the inner and outer perforate which connects the core and bypass passages of the afterburner. The results are discussed in Chapter 6.
4.5. Solution Procedure and software implementation

Figure 4.3: Coupled normalized acoustic pressure and phase spectrum
Chapter 5

Experiments

The actual configuration of afterburner was tested to study the complete behaviour of afterburner with respect to the phenomenon of combustion instability. These tests were also for confirmation of theoretically predicted screech frequencies. The test programs practical implication is that when the instability occurs, the amplitudes of oscillations at various frequencies do not overshoot the permissible limits for the safe afterburner operation with the provision of perforated liner.

In the experiments the duct modes (longitudinal) are excited due to incoming aerodynamic disturbances from the slave combustor, and tangential modes are triggered during afterburner operation. In theoretical analysis, the type of inlet condition that existed in the experiments was not imposed whereas the pressure wave-flame interaction source model which characterizes combustion instability phenomenon in the region of afterburner is considered. As the inlet conditions in the theoretical analysis and the experiments are not the same and since the non-linear effects are not considered in the theoretical analysis, the amplitudes of acoustic oscillations that are observed in the experiments are not directly comparable. However, the frequency spectrum is comparable in predictions and experiments. Thus, while bringing out several aspects of combustion instability from experiments, the predicted modes (frequencies- longitudinal and tangential) are compared with experimental results, to validate theoretical development.

In this chapter, the experimental work carried out is described. First, afterburner test facility description and its capabilities are given. Then, test section preparation along with dynamic pressure probes is given. Finally, the conduction of experiments is presented along with some sample data that was processed through FFT analyser.

5.1 Afterburner test facility

To simulate Afterburner entry conditions, afterburner test facility is employed. The process and instrumentation diagram is shown in figure 5.1. The test facility capabilities are enumerated in Table 5.1. The air is drawn from High mass flow high pressure air supply facility (HMFHPASF) which is located about 50 meters from the Afterburner test facility.
5.1. Afterburner test facility
### 5.1. Afterburner test facility

**Table 5.1: Afterburner test facility capabilities**

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Parameter</th>
<th>Value (Max)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Primary core airflow rate, kg/s</td>
<td>22</td>
<td>Pressurized air supply by a centrifugal compressor</td>
</tr>
<tr>
<td>2.</td>
<td>Bypass air flow rate, kg/s</td>
<td>8</td>
<td>Pressurized air supplied by a compressor</td>
</tr>
<tr>
<td>3.</td>
<td>Primary core flow temperature, K</td>
<td>950</td>
<td>Core air heated by industrial type slave combustor</td>
</tr>
<tr>
<td>4.</td>
<td>Bypass air temperature, K</td>
<td>450</td>
<td>As supplied by the centrifugal compressor</td>
</tr>
<tr>
<td>5.</td>
<td>Number of fuel supply lines</td>
<td>4</td>
<td>Form afterburner fuel system</td>
</tr>
<tr>
<td>6.</td>
<td>Fuel supply injection pressure, kPa</td>
<td>5100</td>
<td>By reheat Dowty fuel pump</td>
</tr>
<tr>
<td>7.</td>
<td>Total afterburner flow rate, LPM</td>
<td>90</td>
<td>By Reheat control system</td>
</tr>
</tbody>
</table>

The slave combustor (pre-heater) shown in figure 5.2, is used to simulate afterburner entry conditions consist of the following major equipments.

1. double shell combustor
2. CPF Type burner with 2 MV atomizer (2 MV is manufacturer’s description of an industrial swirl atomizer in the main burner)
3. Oil Pilot burner with ignition system
4. Flame safe guard system UV 7000
5. Fuel Piping rack train

Air is admitted in to the test facility by opening total isolation valve (TIV). By opening bypass isolation valve (BIV) the total air is bifurcated into two streams in the ratio of 1:5 as the diameter of core line is 400 mm and that of bypass line is 200 mm. By controlling core spill valve (CSV) and core main valve (CMV) required flow through the core is set by fixing the core pressure. Similarly required bypass mass flow rate can be admitted by suitably varying the bypass spill valve (BSV) and bypass control valve (BMV). The flows are measured by vortex flow meters.

The dedicated fuel supply system for the core slave combustor which has two screw type fuel pumps that run at a speed of 450 rpm and develops a pressure of 2430 kPa. First pump is in regular operation. In the event of higher demand of fuel, second pump is activated. Fuel control valve and bypass fuel control valve controls the required amount of fuel supply which is controlled by single loop programmable controller based on 254 kPa differential pressure between atomisation air and fuel line. Fuel flow is measured by turbine flow meter.
5.1. Afterburner test facility

Figure 5.2: The slave combustor
5.2 Afterburner test section

The atomisation air for pilot ignition system and main combustor is supplied by a booster compressor that always operates 500 kPa above the core line pressure. Once the pilot ignition initiates the pilot flame the facility is controlled automatically through burner management system that functions through programmable logic controller (PLC). When the stable flame is observed in the combustor with the help of flame scanner, the plant pressure and temperature is raised suitably. Thus afterburner entry conditions are set.

A separate fuel system is operated for the afterburner operation. Figure 5.3 shows afterburner fuel system. It has four separate fuel lines whose flow is controlled through reheat fuel control unit. It employs Dowty fuel pump to raise the fuel pressure to 5100 kPa. Reheat control unit (RCU) is used with the help of throttle lever to obtain the required flow through the fuel manifolds. Four turbine flow meters are employed for the measurement of fuel flow.

5.2 Afterburner test section

Figure 5.4 shows the test section of 1/3rd scale afterburner test section. The outer casing is made up of stainless steel (SS316). The liner, four fuel manifolds, V-gutter and linkages are made up of nimonic material (GTM-SU-263). Figure 5.5 gives inner details of afterburner which shows, centre body, struts, four fuel manifolds and V-gutter flame stabilizer and liner on afterburner test section. Figure 5.6 shows positioning of screech liner. Figure 5.7 shows test section integrated with test facility.

The dynamic pressure is measured by XTE-190 series Kulite probes [76]. The technical specifications are furnished in Table 5.2.

The probes are flush-mounted on the bypass casing to obtain correct data.
5.2. Afterburner test section

Figure 5.4: Scaled afterburner test section

Figure 5.5: Components of afterburner
5.2. *Afterburner test section*

Figure 5.6: Positioning of screech liner in afterburner test section

Figure 5.7: Test section integrated with test facility with acoustic driver
5.2. Afterburner test section

Table 5.2: Technical data for XTE-190 series Kulite dynamic pressure probes

<table>
<thead>
<tr>
<th>Type</th>
<th>Specification</th>
<th>Value</th>
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</thead>
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<td></td>
<td>Pressure range</td>
<td>690 kPa</td>
</tr>
<tr>
<td>INPUT</td>
<td>Operational mode</td>
<td>Gauge</td>
</tr>
<tr>
<td></td>
<td>Over pressure</td>
<td>3 times rated pressure</td>
</tr>
<tr>
<td></td>
<td>Pressure media</td>
<td>Gases</td>
</tr>
<tr>
<td></td>
<td>Rated electrical excitation</td>
<td>10 VDC/AC</td>
</tr>
<tr>
<td></td>
<td>Maximum electrical Excitation</td>
<td>15 VDC/AC</td>
</tr>
<tr>
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<td>Input impedance</td>
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</tr>
<tr>
<td></td>
<td>Output impedance</td>
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<td></td>
<td>Full scale output (FSO)</td>
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</tr>
<tr>
<td></td>
<td>Residual unbalance</td>
<td>±0.5 % FSO</td>
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<tr>
<td></td>
<td>Combined non-linearity and hysteresis</td>
<td>±0.5 % FS</td>
</tr>
<tr>
<td></td>
<td>Hysteresis</td>
<td>0.1 %</td>
</tr>
<tr>
<td>OUTPUT</td>
<td>Repeatability</td>
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<tr>
<td></td>
<td>Resolution</td>
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<td>Acceleration sensitivity</td>
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<td>Perpendicular</td>
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<td></td>
<td>Transverse</td>
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<tr>
<td></td>
<td>Insulation resistance</td>
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<td></td>
<td>Compensated temperature range</td>
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<td></td>
<td>Thermal zero shift</td>
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<tr>
<td></td>
<td>Thermal sensitivity shift</td>
<td>±2 % /100°F</td>
</tr>
<tr>
<td></td>
<td>Steady acceleration</td>
<td>10,000 g</td>
</tr>
<tr>
<td></td>
<td>Linear vibration</td>
<td>0–2000 Hz sine, 100 g</td>
</tr>
<tr>
<td></td>
<td>Electrical Connection</td>
<td>4 conductor 32 AWG shielded cable 24” long</td>
</tr>
<tr>
<td>PHYSICAL</td>
<td>Electrical Connection</td>
<td>4 conductor 32 AWG shielded cable 24” long</td>
</tr>
<tr>
<td></td>
<td>Weight</td>
<td>5 Grams</td>
</tr>
<tr>
<td></td>
<td>Mounting torque</td>
<td>15 inch pounds</td>
</tr>
<tr>
<td></td>
<td>Sensing principle</td>
<td>Fully active four arm Wheatstone bridge diffused into silicon diaphragm.</td>
</tr>
</tbody>
</table>
5.2. Afterburner test section

The temperature of bypass air during the experiments is in the range of 550 C to 1900 C, which helps in cooling the surface of Kulite dynamic pressure probes. Figure 5.8 shows data acquisition system circuit diagram to obtain dynamic pressure signals. The transducer is flush mounted on the afterbuener casing as shown in figure 5.9. A 10 V DC excitation input voltage is given to each Kulite probe. The output voltage signal which corresponds to dynamic pressure with respect to time is processed through amplifiers and signal conditioners. The data is recorded on a Sony tape recorder. Simultaneously the online data is displayed on a computer monitor through four parallel ports for constant monitoring and to observe the onset of screech and buzz. Online data is displayed in time and frequency domain. Whenever required waterfall diagram is also made available where dynamic pressure amplitude, time and frequency spectrum can be seen. Thereby precautions are taken in the event of undue increase of a dynamic pressure to safe guard the system.

Figure 5.10 shows the circumferential positioning of Kulite probe location in the screech hole region of afterburner test section. The probes on axial direction are limited, as the aim of this work is to capture tangential modes during
5.3. Afterburner test facility operation

When the inlet conditions to the test section are obtained the afterburner fuel is admitted with the help of reheat control unit (RCU) into the manifold that is situated upstream of flame stabilizer. The flame in the test section is initiated when the fuel pressure in the manifold reaches 700 kPa due to the presence of platinum-rhodium catalyst in the recirculation zone of flame stabilizer. Then gradually fuel pressure in the remaining three fuel manifolds is raised to 2720 kPa for full afterburner operation.

Several trails are made to trigger buzz and screech with the test section entry conditions as given in table 5.3. Screech was recorded. The phenomenon was quite violent which is of the order 150 dB; therefore the data was recorded for about 30 seconds. Off-line the data was replayed and analysed [77]. Fourier analysis is carried out to convert the data from time domain to frequency domain to identify the screech frequencies with their amplitudes. The results are discussed in Chapter 6.

Table 5.3: Test conditions

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Core m kg/s</th>
<th>Core p kPa</th>
<th>Core T K</th>
<th>Bypass m kg/s</th>
<th>Bypass p kPa</th>
<th>Bypass T K</th>
<th>Combustor F/A</th>
<th>Overall F/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{a}</td>
<td>6.6</td>
<td>353</td>
<td>873</td>
<td>1.3</td>
<td>396</td>
<td>373</td>
<td>0.020</td>
<td>0.065</td>
</tr>
<tr>
<td>2</td>
<td>6.2</td>
<td>300</td>
<td>856</td>
<td>1.2</td>
<td>337</td>
<td>367</td>
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<td>0.063</td>
</tr>
<tr>
<td>3</td>
<td>5.9</td>
<td>200</td>
<td>840</td>
<td>1.0</td>
<td>224</td>
<td>358</td>
<td>0.018</td>
<td>0.061</td>
</tr>
<tr>
<td>4</td>
<td>5.8</td>
<td>155</td>
<td>820</td>
<td>0.9</td>
<td>174</td>
<td>345</td>
<td>0.017</td>
<td>0.060</td>
</tr>
</tbody>
</table>

\textsuperscript{a} flight condition: ISA +15, M = 0.9, 11 km

afterburner operation.

5.3 Afterburner test facility operation

When the inlet conditions to the test section are obtained the afterburner fuel is admitted with the help of reheat control unit (RCU) into the manifold that is situated upstream of flame stabilizer. The flame in the test section is initiated when the fuel pressure in the manifold reaches 700 kPa due to the presence of platinum-rhodium catalyst in the recirculation zone of flame stabilizer. Then gradually fuel pressure in the remaining three fuel manifolds is raised to 2720 kPa for full afterburner operation.

Several trails are made to trigger buzz and screech with the test section entry conditions as given in table 5.3. Screech was recorded. The phenomenon was quite violent which is of the order 150 dB; therefore the data was recorded for about 30 seconds. Off-line the data was replayed and analysed [77]. Fourier analysis is carried out to convert the data from time domain to frequency domain to identify the screech frequencies with their amplitudes. The results are discussed in Chapter 6.
Chapter 6

Results and Discussion

The results of both the experiments and the computations are presented and discussed in this chapter. The results are discussed under the following subheadings.

1. Frequency characterization of test section
2. Identification of pure and coupled screech modes by analysis
3. Study of effectiveness of screech liner by variation in porosity
4. Attenuation characteristics with 2.5% porosity liner
5. Comparison of theoretical and experimental results.

6.1 Frequency characterization of test section

Figure 6.1 shows the Afterburner test section that was used to conduct tests in the Afterburner test facility. Test results that are presented in this chapter correspond to the test conditions shown in the table 6.1. During testing, core, bypass conditions, combustor fuel to air ratio and overall fuel to air ratio (Combustor and afterburner) are simulated.

Figure 6.2 shows the afterburner configuration that was modelled for FEM acoustic analysis. The acoustic mesh is modelled from the slave combustor exit onwards. Centre body, symmetrical struts and flame stabiliser are taken into account in meshing. Bypass air column is provided where the perforate liner is positioned. The software limits the use of different core and bypass conditions, by which it means fluid properties for bypass acoustic medium are same as that of core. Hence for core and bypass, conditions as shown in table 6.1 are used for analysis.

The frequency spectrum for the afterburner for a range from 10 Hz to 2000 Hz is predicted. The normalized acoustic pressure in dB and corresponding phase in degrees are plotted in the figure 6.3. The analysis is carried out with an inlet excitation particle velocity of $1 + 0j$ m/s. This value was chosen to be consistent with several earlier researchers in the field [78, 27]. Hence, this magnitude will result in the prediction of acoustic pressure in the range of 150 dB to 220 dB.
6.1. Frequency characterization of test section

Figure 6.1: Tested Afterburner configuration

Table 6.1: Test conditions for experiments and analysis

<table>
<thead>
<tr>
<th>Experimental condition</th>
<th>Core</th>
<th>Bypass</th>
<th>Combustor</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sl. No.</td>
<td>( \dot{m} ) kg/s</td>
<td>( p ) kPa</td>
<td>( T ) K</td>
<td>( \dot{m} ) kg/s</td>
</tr>
<tr>
<td>1</td>
<td>6.6</td>
<td>353</td>
<td>873</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Conditions for computational studies

<table>
<thead>
<tr>
<th>Core</th>
<th>Bypass</th>
<th>Combustor</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sl. No.</td>
<td>( \dot{m} ) kg/s</td>
<td>( p ) kPa</td>
<td>( T ) K</td>
</tr>
<tr>
<td>1</td>
<td>6.6</td>
<td>353</td>
<td>873</td>
</tr>
</tbody>
</table>

Figure 6.2: Analysed afterburner configuration
6.1. Frequency characterization of test section

Since the equations solved are linear, the resulting magnitude is proportional to the input and if normalized with the input amplitude the results would be invariant with the imposed amplitude. Nozzle admittance that describes the choked condition is imposed at the entry to the convergent nozzle. It is noticed that only the longitudinal acoustic modes are excited within the test section. Each mode is identified by the phase shift of $180^\circ$. Table 6.2 gives the summary of all the frequencies by prediction and experiments.

Figure 6.4 is the prediction of frequency spectrum for the afterburner for a frequency range from 10 Hz to 2000 Hz with heat release. The normalized acoustic pressure in dB and corresponding phase shift in degrees are plotted. The analysis is carried out with the same boundary conditions as enumerated for figure 6.3 with the coupling of heat release acoustic oscillations. It is noticed that along with longitudinal acoustic modes, three additional tangential modes are excited within the test section. Each mode is identified by the phase shift of $180^\circ$. In both the plots, the cut-on frequencies correspond to the peaks and are clearly visible.

Figure 6.5 is the experimentally obtained frequency spectrum of Afterburner with heat addition. In the experiments, longitudinal instrumentation is limited. However, all the peak frequencies that are observed in the prediction and experiments are tabulated in table 6.2. Since eight probes are provisioned in the circumferential cross section of afterburner it was possible to identify the tangential modes by their respective phase shift. In the event of gas flowing in the test section the cut-on frequency changes. The effective cut-on frequency will vary with flow velocity as

$$f' = f \sqrt{1 - M^2}$$  \hspace{1cm} (6.1)

where $f$ is the original cut-on frequency without flow. Thus from the experimentally obtained frequency data, frequency that corresponds to no flow condition...
6.1. Frequency characterization of test section

Figure 6.4: Predicted frequency spectrum of afterburner with heat release

Figure 6.5: Experimentally obtained frequency spectrum of afterburner
Table 6.2: Theoretical and experimental frequencies

<table>
<thead>
<tr>
<th>Sl. No.</th>
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<th>Experiment</th>
<th>Deviation (%)</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
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<td>Without flow (M = 0.3)</td>
<td>With mean flow</td>
<td>Without mean flow</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Without heat rel</td>
<td>With heat rel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>120</td>
<td>170</td>
<td>148</td>
<td>155</td>
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<td>610</td>
<td>644</td>
<td>677</td>
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<tr>
<td>4</td>
<td>840</td>
<td>840</td>
<td>743</td>
<td>778</td>
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<tr>
<td>5</td>
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<td>12</td>
<td>–</td>
<td>1840</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

$^a$ The mode is determined from theory as the longitudinal instrumentation is limited.

The deviation that was observed in the prediction and experiments in respect of longitudinal modes can be viewed in the following way. Consider fluid in a circular pipe which is driven by a vibrating piston and terminated in acoustic impedance and a resonant frequency defined as that at which the reactive component of the input impedance vanishes. At this frequency the input impedance is a minimum and the power radiated out of an open ended pipe is a maximum [79]. Then the first longitudinal mode of an open ended pipe will be 242 Hz and for a pipe closed by rigid cap the frequency is 129 Hz. Therefore, for actual case, it is expected that the first longitudinal mode lies in between these frequencies. Three-dimensional analysis predicted first mode as 120 Hz. This is because of the fact of centre body, struts and flame stabilizer along with choked nozzle admittance. With heat release the first longitudinal mode predicted by software is 170 Hz in the absence of mean gas motion. This shift in frequency is expected due to heat release rate imposed in the afterburner region, as the temperature effect changes the sound velocity. With gas motion, and afterburner in operation the first frequency encountered in the experiments is 148 Hz, and that with out the gas motion is 155 Hz, having a deviation of 8.8 %. Similar exercise is carried out for all the longitudinal frequencies and their deviations are reported in the table. The maximum deviation observed in this work is less than 10 %. Bloxidge [10], while investigating low frequency longitudinal oscillations...
observed about 7% deviation between theory and experiments. This level of deviation is expected due to limitations in software implementations such as fluid properties of acoustic medium and the compressible effects that arise due to flow Mach number increasing beyond 0.3 at the centre body region of the test section.

In the case of tangential cut-on modes, two cases arise due to the fact that the geometry of the test section is a simple tube upstream and downstream of the centre body, and in the centre body region the acoustic behaviour is governed by annular passage. The centre body introduces another complexity for tangential mode calculation in annular passage because of gradually varying nose at upstream and a diffuser cross section downstream. Tangential modes for a tube is given by the boundary condition that the radial particle velocity at the walls is zero. Accordingly, \( k_r R = 0, 1.84, 3.05, 3.83, \ldots \) etc. These values are the solution of the equation \( J_m'(k_r R) = 0 \). So, for the test section considered, the first tangential mode at upstream of centre body is 713 Hz and at downstream is 675 Hz. But it is observed that neither prediction nor experiments, have captured these two frequencies. Therefore, the cut-on frequencies due to annulus passage are examined. They are obtained by seeking a solution to the following equation

\[
J_m'(k_r R_1)Y_m'(k_r R_2) - j_m'(k_r R_2)Y_m'(k_r R_1) = 0 \quad (6.2)
\]

The cut-on tangential frequencies are the roots of the above equation. Thus the first tangential mode is calculated as 976 Hz. The software predicts as 1081 Hz, having a deviation of about 9.7%. This deviation can be ascribed to the non-uniform centre body along with other reasons cited above for low frequency predictions. The software prediction is compared with experimentally obtained value in table 6.2, which shows a deviation of 7.7%. Further reason for deviation in frequencies is due to temperature gradient in the afterburner as the fuel droplets burn gradually across the axial length of the combustion chamber. Due to software limitation it was not possible to implement the temperature variation in the prediction. It may be noted here, that a temperature rise (from burnt region to unburnt region) ratio, if it is of the order 2, will cause a frequency shift by up to 20% [34]. The main conclusion that could be drawn is that the annulus area due to centre body is profoundly dictating the commencement of first cut-on tangential mode, rather than the simple extended tube portions of the test section at upstream and downstream of centre body. This fact is confirmed by both the theory and experiments. On similar lines, Bloxidge et al [10] and Dowling [11] and have observed that the centre body could be the potential acoustic driver.

It is not straightforward to discuss about the coupled modes viz., (0,1,1) and (0,1,2) that are found in prediction. At present the only explanation that could be offered is that the kind of mode coupling — first and second longitudinal with first tangential mode — is due to the cross sectional area variation of afterburner [49]. In the experiments the second tangential mode could be called as coupled mode by the conclusion drawn from theory. But firm confirmation is possible only by providing instrumentation in longitudinal direction and by taking their cross spectra.
6.2 Identification of pure and coupled screech modes by analysis

Figure 6.6, figure 6.7 and figure 6.8 show the acoustic pressure history of afterburner with heat release for first three longitudinal modes. Figure 6.9, figure 6.10 and figure 6.11 show the acoustic pressure history of afterburner with heat release for first three longitudinal modes. From the pressure history three tangential screech modes are observed. They are identified as: i) mode(0,1,0), \( f = 1081 \) Hz, ii) mode(0,1,1), \( f = 1288 \) Hz and iii) mode(0,1,2), \( f = 1440 \) Hz. First one is a pure screech mode having single nodal line along the entire axial length, shown in figure 6.9. The second one is a coupled mode—coupling taking place between first screech mode and first longitudinal mode. This is revealed by the acoustic
6.2. Identification of pure and coupled screech modes by analysis

Figure 6.9: Predicted acoustic pressure history for (0,1,0) mode

Figure 6.10: Predicted acoustic pressure history for (0,1,1) coupled mode

Figure 6.11: Predicted acoustic pressure history for (0,1,2) coupled mode
6.3. Study of effectiveness of screech liner by variation in porosity

Figure 6.12: Identification of front and back plane on Afterburner

Pressure variation depicted in figure 6.10, having single nodal line along the entire length of the test section and another nodal line in the middle of the test section. The third one is again a coupled mode—coupling taking place between first tangential mode and second longitudinal mode which is shown in figure 6.11. The strong coupling is due to the centre body inside the test section that creates a variable area along the cross-section of test section. Although the first screech mode is cut on in the variable area region due to centre body at the centre of the test section, its effect at the beginning of afterburner entry is strongly felt.

To gain further understanding into tangential modes that were identified by the above figures, it is expedient to draw cross sectional variation of acoustic pressure in two planes of the test section. For this purpose a front plane and back plane are marked on the test section as shown in figure 6.12. The front plane is immediately after the flame stabilizer and back plane is chosen at the middle of the test section.

Figure 6.13 and figure 6.14 shows the variation of acoustic pressure in the cross sectional plane for front and back plane. Three tangential modes are compared. It is observed that first pure mode absorbs more acoustic energy than the coupled modes. The trend is maintained throughout the test section. In figure 6.15, figure 6.16 and figure 6.17 the acoustic pressure at front and back planes are compared for $f = 1081 \text{ Hz}$, $f = 1288 \text{ Hz}$ and $f = 1440 \text{ Hz}$ separately. It is observed that all of them seem to be propagating modes without any reduction in amplitude from flame stabilizer to the end of test section.

6.3 Study of effectiveness of screech liner by variation in porosity

Parametric study of screech liner with different porosity is presented in the following figures. Two extreme cases are studied. At bypass entry and exit open end tube boundary condition is imposed by specifying acoustic pressure to be
6.3. Study of effectiveness of screech liner by variation in porosity

Figure 6.13: Normalized acoustic pressure variation at front plane of Afterburner

Figure 6.14: Normalized acoustic pressure variation at back plane of Afterburner
6.3. Study of effectiveness of screech liner by variation in porosity

Figure 6.15: Comparison of normalized acoustic pressure variation at front and back plane of Afterburner for \( f = 1081 \) Hz

Figure 6.16: Comparison of normalized acoustic pressure variation at front and back plane of Afterburner for \( f = 1288 \) Hz
6.4. Attenuation characteristics with 2.5 % porosity liner

The results showed large attenuation due to perforate liner. Secondly, rigid boundary condition is assumed at entry and exit of bypass. The foregoing results are for the second case. In actual experimental set up, the boundary condition has to be in between rigid boundary and open tube, because bypass air enters the core region at several other places for film cooling purposes. So, the case presented corresponds to highest amplitude due to combustion source, to bring out the effectiveness of screech liner profoundly. In figure 6.18 to figure 6.23 a comparison of variation of normalized acoustic pressure along two cross-sections of afterburner for screech liner porosity varying from 1.5 % to 4 % is presented.

At front plane, for \( f = 1081 \) Hz, a porosity factor of 1.5 % does not show appreciable attenuation. Where as with 2.5 % porosity significant attenuation is noticed, but with 4 % porosity, the gain is very minimal. Similar trends are visible for \( f =1288 \) Hz and \( f = 1440 \) Hz, both at front plane and back plane. Thus, it was observed that 2.5 % porosity gives maximum advantage in respect of screech liner. Hence, it was decided to perform analysis and experiments with the perforate screech liner with the porosity of 2.5 %.

**6.4 Attenuation characteristics with 2.5 % porosity liner**

Figure 6.24 shows a comparison of normalized acoustic pressure with and without the perforate liner. The effect of having porous liner with a factor of 2.5 % is compared with that of no liner. The plot shows the attenuation characteristics for tangential modes. The relative attenuation values are shown in the table 6.3. It is observed that in general there is a considerable attenuation achieved due
6.4. Attenuation characteristics with 2.5 % porosity liner

Figure 6.18: Comparison of normalized acoustic pressure variation for different porosity at front plane of Afterburner for \( f = 1081 \) Hz

Figure 6.19: Comparison of normalized acoustic pressure variation for different porosity at front plane of Afterburner for \( f = 1288 \) Hz
6.4. Attenuation characteristics with 2.5% porosity liner

Figure 6.20: Comparison of normalized acoustic pressure variation for different porosity at front plane of Afterburner for $f = 1440$ Hz

Figure 6.21: Comparison of normalized acoustic pressure variation for different porosity at back plane of Afterburner for $f = 1081$ Hz
6.4. Attenuation characteristics with 2.5 % porosity liner

Figure 6.22: Comparison of normalized acoustic pressure variation for different porosity at back plane of Afterburner for $f = 1288$ Hz

Figure 6.23: Comparison of normalized acoustic pressure variation for different porosity at back plane of Afterburner for $f = 1440$ Hz
6.4. Attenuation characteristics with 2.5% porosity liner

Figure 6.24: Comparison of acoustic pressure with and without perforate for screech frequencies

Table 6.3: Effect of perforate liner on the attenuation characteristics of the afterburner with heat release

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Frequency (Hz)</th>
<th>Acoustic pressure (dB)</th>
<th>% ge reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Without perforate</td>
<td>With Perforate</td>
</tr>
<tr>
<td>1</td>
<td>1081</td>
<td>244</td>
<td>212</td>
</tr>
<tr>
<td>2</td>
<td>1288</td>
<td>228</td>
<td>206</td>
</tr>
<tr>
<td>3</td>
<td>1440</td>
<td>220</td>
<td>201</td>
</tr>
</tbody>
</table>
6.4. Attenuation characteristics with 2.5 \% porosity liner

Figure 6.25: Comparison of acoustic pressure with and without perforate in the front plane for \( f = 1081 \) Hz

Figure 6.26: Comparison of acoustic pressure with and without perforate in the front plane for \( f = 1288 \) Hz
6.4. Attenuation characteristics with 2.5% porosity liner

Figure 6.27: Comparison of acoustic pressure with and without perforate in the front plane for \( f = 1440 \) Hz

Figure 6.28: Comparison of acoustic pressure with and without perforate in the back plane for \( f = 1081 \) Hz
6.4. Attenuation characteristics with 2.5 % porosity liner

Figure 6.29: Comparison of acoustic pressure with and without perforate in the back plane for $f = 1288$ Hz

Figure 6.30: Comparison of acoustic pressure with and without perforate in the back plane for $f = 1440$ Hz
6.5 Comparison of theoretical and experimental results

Figure 6.31: Acoustic pressure amplitude and phase with perforate liner at 0° probe location

to the presence of porous liner. Greatest attenuation is seen in the case of first tangential mode ($f = 1081$ Hz). In figure 6.25 to figure 6.30 a comparison of each tangential mode is presented for both in the front and back plane. It is observed that there is a remarkable attenuation achieved due to porous liner.

6.5 Comparison of theoretical and experimental results

Figure 6.31 to figure 6.34 gives the data obtained from the rig runs, which were conducted for confirmation of screech mode identification. The first pure screech tangential mode and second screech coupled tangential modes are captured. There was a phase damping to the tune of $50^\circ$ noticed in the experiments for the second mode and more than $50^\circ$ in the case of first mode. Here, the term phase damping implies a phase shift which is obtained when cross spectrum of each probe is taken with a reference probe. This type of shift is specifically observed in experiments due to temperature gradient in the tangential direction in the cross sectional plane of the afterburner, where eight probes are located. In the prediction $180^\circ$ phase shift is obtained due to the assumption of uniform temperature in the entire acoustic domain. It is observed that at $180^\circ$ probe location the amplitude of first mode is higher than the second mode as predicted. For clarity acoustic pressure and Phase are shown in figure 6.35 and figure 6.36 from the rig testing.

Details of the experimental results are tabulated in table 6.4 in which at four probe locations, tangential modes frequency, amplitude, and phase are shown. Phase damping is observed in the experiments.

Figure 6.37 shows the acoustic pressure and phase for three tangential modes
6.5. Comparison of theoretical and experimental results

Figure 6.32: Acoustic pressure amplitude and phase with perforate liner at 90° probe location

Figure 6.33: Acoustic pressure amplitude and phase with perforate liner at 180° probe location
6.5. Comparison of theoretical and experimental results

Figure 6.34: Acoustic pressure amplitude and phase with perforate liner at 270° probe location

Figure 6.35: Acoustic pressure amplitude with perforate liner at 180° probe location
6.5. Comparison of theoretical and experimental results

Figure 6.36: Acoustic pressure phase with perforate liner at 180° probe location

Table 6.4: Experimental results of screech modes

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Probe location (degree)</th>
<th>Frequency (Hz)</th>
<th>Amplitude (Pa)</th>
<th>Phase (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First screech tangential mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>947</td>
<td>160</td>
<td>96</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>947</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>947</td>
<td>267</td>
<td>126</td>
</tr>
<tr>
<td>4</td>
<td>270</td>
<td>947</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Second screech coupled tangential mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1294</td>
<td>337</td>
<td>130</td>
</tr>
<tr>
<td>2</td>
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<td>0</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>1294</td>
<td>214</td>
<td>130</td>
</tr>
<tr>
<td>4</td>
<td>270</td>
<td>1294</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
6.5. Comparison of theoretical and experimental results

Figure 6.37: Predicted acoustic pressure amplitude and phase with perforate liner

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Frequency (Hz)</th>
<th>Phase (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1081</td>
<td>180</td>
</tr>
<tr>
<td>2</td>
<td>1288</td>
<td>180</td>
</tr>
<tr>
<td>3</td>
<td>1440</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 6.5: Theoretical results for afterburner at screech frequencies

from predictions. Table 6.5 gives details of theoretical estimates. Only frequency and phase are tabulated. Referring to table 6.4 and 6.5, the theoretical frequencies for first and second tangential modes with their phases are comparable with experimental results. Though third tangential mode is predicted, it was not excited in the experiments.

There was certain level of deviation in the prediction of these frequencies, when compared to the experimentally obtained values. The reasons for such a shift in frequency may be sighted as: i) in the analysis, the acoustic medium of core and bypass are treated with same acoustic properties as core; ii) mean flow velocities of core and bypass could not be incorporated in the FEM analysis; iii) during the experiments the sound velocity is not uniform due to afterburner combustion though for a short period, which is not accounted in the analysis. The corresponding amplitudes for these frequencies are not shown for comparison, because of the fact that the inlet excitation in the theory is not same as the experiments. It may be recalled that in the analysis, the inlet excitation is by harmonic particle velocity and in the experiments the pre-heater combustor, high speed turbulent stream provides the inlet excitation. Both the analysis and experiments confirmed that the screech modes do not get excited without the combustion source in the afterburner region. For this test section of length to
6.5. Comparison of theoretical and experimental results

diameter ratio of 5, no radial modes are encountered both in the analysis and experiments in the frequency range of interest.
Chapter 7

Conclusions

The screech combustion instability in afterburner is analysed by considering variable area cross section, and modelling of combustion source acoustic oscillations. Theoretical estimates are carried out by a three dimensional finite element analysis by considering the combustion flame oscillations and transfer relation of perforate liner. The actual configuration of afterburner was tested.

The governing equations for the acoustic response of the afterburner are derived with the boundary conditions, both at inlet and outlet of test sections, acoustic impedance boundary condition that characterizes the screech liner and exit nozzle admittance. The mathematical treatment is presented for completion.

The most important aspect related to combustion instability in afterburners is of pressure wave-flame interaction. In this work combustion flame response is developed from fundamentals which is unique compared to the body of literature on simple conventional flame response to pressure disturbances of different amplitudes characterized by different length scales. The same is implemented as a user defined routine source function in the SYSNOISE software.

The actual configuration of afterburner was tested to study the complete behaviour of afterburner with respect to the phenomenon of combustion instability. These tests were also for confirmation of theoretically predicted screech frequencies. The test programs practical implication is that when the instability occurs, the amplitudes of oscillations at various frequencies do not overshoot the permissible limits for the safe afterburner operation with the provision of perforated liner.

In the experiments the duct modes (longitudinal) are excited due to incoming aerodynamic disturbances from the slave combustor, and tangential modes are triggered during afterburner operation. In theoretical analysis, the type of inlet condition that existed in the experiments was not imposed whereas the pressure wave-flame interaction source model which characterizes combustion instability phenomenon in the region of afterburner is considered. As the inlet conditions in the theoretical analysis and the experiments are not the same and since the non-linear effects are not considered in the theoretical analysis, the amplitudes of acoustic oscillations that are observed in the experiments are not directly comparable. However, the frequency spectrum is comparable in predictions and experiments. Thus, while bringing out several aspects of combustion instability
from experiments, the predicted modes (frequencies- longitudinal and tangential) are compared with experimental results, to validate theoretical development. The following conclusions are drawn:

1. Without the heat source only longitudinal acoustic modes are found to be excited in the afterburner test section. With heat release, three additional tangential modes are excited. Each mode is identified by the phase shift of $180^\circ$.

2. Since eight probes are provisioned in the circumferential cross section of afterburner it was possible to identify the tangential modes by their respective phase shift in the experiments.

3. From the pressure history three tangential screech modes are clearly identified.

4. Two coupled tangential modes resulted due to variable area of the afterburner. It is observed that first pure mode absorbs more acoustic energy than the coupled modes. It is also observed that away from the flame stabilizer second coupled tangential mode shows reduction in amplitude.

5. Parametric study of screech liner porosity factor of 1.5 % has not shown appreciable attenuation. Where as with 2.5 % porosity significant attenuation is noticed, but with 4 % porosity, the gain is very minimal. Hence, the perforate screech liner with the porosity of 2.5 % is finalized. Greatest attenuation is seen in the case of first tangential mode.

6. From the rig runs, first pure screech tangential mode and second screech coupled tangential modes are captured. There was a phase damping to the tune of $50^\circ$ noticed in the experiments for the second mode and more than $50^\circ$ in the case of first mode. It is observed that at $180^\circ$ probe location the amplitude of first mode is higher than the second mode as predicted.

7. The theoretical frequencies for first and second tangential modes with their phases are comparable with experimental results. Though third tangential mode is predicted, it was not excited in the experiments. There was certain level of deviation in the prediction of these frequencies, when compared to the experimentally obtained values.

8. For this test section of length to diameter ratio of 5, no radial modes are encountered both in the analysis and experiments in the frequency range of interest.

9. The design inputs of screech liner are implemented in the actual engine afterburner. The tests of afterburner in the altitude standalone test facility have shown that the combustion instabilities amplitudes are with in the permissible limits.

10. The exercise provides the expertise to analyse screech related problems due to combustion source. When the experiments are quite difficult and costly
the analysis becomes the guideline for the confirmatory testing of afterburners. It may be noted that triggering screech phenomenon itself is violent to the tune of 150 dB and data cannot be logged in for a longer duration, because when screech modes are cut-on, the amplitudes grow exponentially leading to structural failure. For this reason, theoretical estimates provide a roadmap in the experiments to identify the screech modes quickly by on-line processing the data though for a short duration of the order 30 seconds or so.

In summary, an acoustic model has been developed for the afterburner combustor, taking into account the combustion response, the screech liner and the nozzle to study the acoustic instability of the afterburner. The flame response model has been validated experimentally for longitudinal and tangential frequencies using a model test rig and the results have given sufficient confidence to apply the model for full scale afterburners as a predictive design tool. The absolute amplitudes cannot be predicted using the linear analysis presented here, which does not take into account the saturation effects. Further, the damping effects, such as droplet damping and viscous damping, are neglected in the analysis. The future scope of work could be through CFD unsteady analysis where effect of many of these parameters could be evaluated and the magnitude effects can also be predicted.
Appendix A

Nozzle Admittance

The theory of nozzle admittance was proposed by Crocco [68] in 1960 for rocket engines. This theory is adopted for the present work also. The analysis is reproduced here for the sake of completeness.

A.1 Nozzle admittance for choked convergent nozzle

The behaviour of nozzle for non-isentropic, non-isothermal in the range of oscillatory frequencies is given by using the continuity, momentum and energy equations. Assuming the perturbations, \( p', \rho', \) and \( u' \) to be small compared with the unperturbed quantities, the governing equations can be written in the following form, retaining only the first order terms in the perturbations.

\[
\frac{\partial}{\partial t^*} \left( \frac{\rho'}{\bar{\rho}} \right) + \bar{u} \frac{\partial}{\partial z^*} \left( \frac{\rho'}{\bar{\rho}} + \frac{u'}{\bar{u}} \right) = 0 \tag{A.1}
\]

\[
\frac{\partial}{\partial t^*} \left( \frac{u'}{\bar{u}} \right) + \left( \frac{\rho'}{\bar{\rho}} + 2 \frac{u'}{\bar{u}} \right) \frac{d\bar{u}}{dz^*} + \bar{u} \frac{\partial}{\partial z^*} \left( \frac{u'}{\bar{u}} \right) = \frac{p'}{\bar{p}} \frac{d\bar{u}}{dz^*} - \frac{\bar{p}}{\bar{\rho}u} \frac{\partial}{\partial z^*} \left( \frac{p'}{\bar{p}} \right) \tag{A.2}
\]

\[
\left( \frac{\partial}{\partial t^*} + \bar{u} \frac{\partial}{\partial z^*} \right) \frac{s'}{c_v} = \left( \frac{\partial}{\partial t^*} + \bar{u} \frac{\partial}{\partial z^*} \right) \left( \frac{p'}{\bar{p}} - \gamma \frac{\rho'}{\bar{\rho}} \right) = 0 \tag{A.3}
\]

where \( z^* \) is the distance along the nozzle and \( t^* \) is the time. The energy equation is written expressing the constancy of entropy where \( s' \) is the entropy perturbation, \( c_v \) is the constant volume specific heat, and \( \gamma \) the adiabatic index.

In these equations \( \bar{u}, d\bar{u}/dz^* \) and \( \bar{p}/\bar{\rho} \) are known functions of \( z^* \) determined by the nozzle shape. Due to the linearity of the three equations above, the harmonic form of oscillatory time dependence is chosen and, using the complex representation, the dependent variables are written as

\[
p'/\bar{p} = \varphi(z^*)e^{j\omega t^*}; \quad \rho'/\bar{\rho} = \sigma(z^*)e^{j\omega t^*}; \quad u'/\bar{u} = \nu(z^*)e^{j\omega t^*} \tag{A.4}
\]

where \( \omega \) is the angular frequency of oscillation and \( \varphi, \sigma \) and \( \nu \) are complex functions of \( z^* \) alone. At the nozzle entrance, \( z^* = z_e^* \), the three functions have certain values \( \varphi_e, \sigma_e, \nu_e \). The boundary condition to be applied to the rest of the flow system as a result of the presence of nozzle is determined by the relations between \( \varphi_e, \sigma_e, \nu_e \).
Equation A.3 is integrated as

\[
\frac{s'}{c_v} = \left( \frac{p'}{\bar{p}} - \gamma \frac{\rho'}{\bar{\rho}} \right) = \psi \left( t^* - \int_{z_e^*}^{z^*} \frac{dz^*}{\bar{u}} \right)
\] (A.5)

where arbitrary function \( \psi \) being in general determined by the known time dependence of the entropy at \( z^* = z_e^* \). For the exponential time dependence assumed in equations A.4

\[
\varphi(z^*) - \gamma \sigma(z^*) = \varepsilon \exp \left( -j\omega \int_{z_e^*}^{z^*} \frac{dz^*}{\bar{u}} \right)
\] (A.6)

where the constant \( \varepsilon \) represents the amplitude of the entropy oscillation divided by \( c_v \). With the assumption A.4 and the relation A.5, equations A.1 and A.2 are reduced to the following system of ordinary differential equations in \( \nu \) and \( \sigma \).

\[
\bar{u} \frac{d\nu}{dz^*} + \bar{u} \frac{d\sigma}{dz^*} + j\omega \sigma = 0
\] (A.7)

\[
\bar{u} \frac{d\nu}{dz^*} + \frac{c_v^2}{\bar{u}} \frac{d\sigma}{dz^*} + \left( \frac{j\bar{u}}{\bar{\bar{u}}} + j\omega \right) \nu - (\gamma - 1) \frac{d\bar{u}}{dz^*} \sigma = \varepsilon \left( \frac{d\bar{u}}{dz^*} + j\omega \frac{c_v^2}{\gamma \bar{u}^2} \right) \exp \left( -j\omega \int_{z_e^*}^{z^*} \frac{dz^*}{\bar{u}} \right)
\] (A.8)

An analytical solution is obtained by confining our domain to nozzle in which \( \bar{u} \) increases linearly with \( z^* \) in the subsonic portion of the nozzle. Therefore,

\[
\frac{d\bar{u}}{dz^*} = \frac{\bar{u}}{z_e^*} \frac{c_v^*}{\bar{u}_e} \frac{c_v^* - \bar{u}_e}{l_{sub} \cdot L}
\] (A.9)

in which \( c_v^* \) represents the critical sound speed attained at the throat where \( z^* = z_e^* \), \( l_{sub} = (z_e^* - z_e^*)/L \) represents the ratio of the length of the subsonic portion of the nozzle to the chamber length \( L \). The subscript * is used to denote conditions at the nozzle throat.

Moreover a new independent variable is defined as

\[
z = \left( z^* / z_e^* \right)^2 = \left( \bar{u} / c_v^* \right)^2
\] (A.10)

in terms of which

\[
c_v^2 = c_v^* \cdot \left\{ \frac{1}{2} (\gamma + 1) - \frac{1}{2} (\gamma - 1) z \right\}
\] (A.11)

\[
\int_{z_e^*}^{z^*} \frac{dz^*}{\bar{u}} = \frac{z_e^*}{2c_v^*} \log \frac{z}{z_e^*}
\] (A.12)

where \( z_e \) represents the assigned value of \( z \) at the nozzle entrance; and a reduced angular frequency is defined as

\[
\beta = \frac{z_e^* \cdot \omega}{c_v^*} = \frac{\omega l_{sub} \cdot L}{c_v^* - \bar{u}_e}
\] (A.13)
Introducing equations A.9 and A.13 into equations A.7 and A.8 and eliminating \( dv/dz^* \) and by eliminating \( \nu \) a non-homogeneous complex hypergeometric equation is derived. The solution procedure is adopted from Crocco [68]. Finally, admittance boundary condition is computed that can be applied at the entrance of nozzle is given by the procedure by Crocco [68] for the high frequencies in the non-isothermal case

\[
\frac{\nu_e}{\sigma_e} = \frac{\eta^{(0)}}{\gamma} + \frac{1}{i\beta} \left( \frac{1 - \gamma}{\gamma} \frac{\eta^{(0)}}{\frac{\gamma+1}{2} z - \frac{\gamma-1}{2}} + \frac{\eta^{(1)}}{\gamma} \right) \quad \text{for } \beta \gg 1 \tag{A.14}
\]

where

\[
\eta^{(0)} = \left\{ 1 + \frac{1}{2(\gamma + 1)(1 - z)/z} \right\}^{1/2}
\]

\[
\eta^{(1)} = \gamma - 1 - 2\eta^{(0)} - (\gamma + 1)(1 - z)y^{(1)}
\]

\[
y^{(1)} = \frac{1}{2 \left\{ 1 + \frac{1}{2(\gamma + 1)(1 - z)/z} \right\}^{1/2}} \left[ (1 - z) \frac{dy^{(0)}}{dz} - 2y^{(0)} - \frac{1}{\gamma + 1 \frac{1}{z}} \right]
\]

\[
y^{(0)} = \frac{1}{(\gamma + 1)(1 - z)} \left( \sqrt{1 + \frac{\gamma + 1}{2} - \frac{1}{z}} \right)
\]

\[
\beta = \frac{z_\ast 2\pi f}{c_\ast}
\]

in which \( c_\ast \) represents the critical sound speed attained at the throat where \( z = z_\ast \).
Bibliography


